Lecture-11

Synthesizing Realistic Facial Expressions from Photographs:
Pighin et al
SIGGRAPH'98

Synthesizing Realistic Facial Expressions from Photographs

Pighin et al

SIGGRAPH'98

The Artist's Complete Guide to Facial Expression: Gary Faigin

- There is no landscape that we know as well as the human face. The twenty-five-odd square inches containing the features is the most intimately scrutinized piece of territory in existence, examined constantly, and carefully, with far more than an intellectual interest. Every detail of the nose, eyes, and mouth, every regularity in proportion, every variation from one individual to the next, are matters about which we are all authorities.

Main Points

- One view is not enough.
- Fitting of wire frame model to the image is a complex problem (pose estimation)
- Texture mapping is an important problem

Synthesizing Realistic Facial Expressions

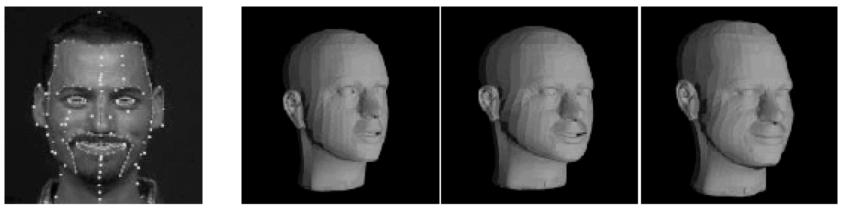
- Select 13 feature points manually in face image corresponding to points in face model created with Alias.
- Estimate camera poses and deformed 3d model points.
- Use these deformed values to deform the remaining points on the mesh using interpolation.

Synthesizing Realistic Facial Expressions

- Introduce more feature points (99) manually, and compute deformations as before by keeping the camera poses fixed.
- Use these deformed values to deform the remaining points on the mesh using interpolation as before.
- Extract texture.
- Create new expressions using morphing.

Synthesizing Realistic Facial Expressions

(a)



3D Rigid Transformation

$$\begin{bmatrix}X\\Y\\Z\end{bmatrix} = R\begin{bmatrix}X\\Y\\Z\end{bmatrix} + T = \begin{bmatrix}r_{11} & r_{12} & r_{13}\\ r_{21} & r_{22} & r_{23}\\ r_{31} & r_{32} & r_{33}\end{bmatrix}\begin{bmatrix}X\\Y\\Z\end{bmatrix} + \begin{bmatrix}T_X\\T_Y\\T_Z\end{bmatrix}$$

Camera coordinates

Wireframe coordinates

$$x_i^k = f^k \frac{X_i^k}{Z_i^k}, \quad y_i^k = f^k \frac{Y_i^k}{Z_i^k}$$ perspective

K=camera no.

3D Rigid Transformation

$$x_i^k = f^k \frac{X_i^k}{Z_i^k}, \quad y_i^k = f^k \frac{Y_i^k}{Z_i^k}$$

$$x_i^k = f_k \frac{r_{11}^k X_i + r_{12}^k Y_i + r_{13}^k Z_i + T_X^k}{r_{31}^k X_i + r_{32}^k Y_i + r_{33}^k Z_i + T_Z^k}$$

$$y_i^k = f_k \frac{r_{21}^k X_i + r_{22}^k Y_i + r_{23}^k Z_i + T_Y^k}{r_{31}^k X_i + r_{32}^k Y_i + r_{33}^k Z_i + T_Z^k}$$

Model Fitting

$$x_i^k = f_k \frac{\mathbf{r}_x^{\mathbf{k}} \mathbf{p}_\mathbf{i} + T_X^k}{\mathbf{r}_z^{\mathbf{k}} \mathbf{p}_\mathbf{i} + T_Z^k}$$

$$y_i^k = f_k \frac{\mathbf{r}_y^{\mathbf{k}} \mathbf{p}_\mathbf{i} + T_Y^k}{\mathbf{r}_z^{\mathbf{k}} \mathbf{p}_\mathbf{i} + T_Z^k}$$

Model Fitting

$$x_i'^k = f_k \frac{\mathbf{r}_x^{\mathbf{k}} \mathbf{p}_\mathbf{i} + T_X^k}{\mathbf{r}_z^{\mathbf{k}} \mathbf{p}_\mathbf{i} + T_Z^k}$$

$$y_i'^k = f_k \frac{\mathbf{r}_y^{\mathbf{k}} \mathbf{p}_\mathbf{i} + T_Y^k}{\mathbf{r}_z^{\mathbf{k}} \mathbf{p}_\mathbf{i} + T_Z^k} \qquad \eta^k = \frac{1}{T_Z^k}, s^k = f^k \eta^k$$

$$x_i'^k = s_k \frac{\mathbf{r}_x^{\mathbf{k}} \mathbf{p}_\mathbf{i} + T_X^k}{1 + \eta^k \mathbf{r}_z^{\mathbf{k}} \mathbf{p}_\mathbf{i}}$$

$$y_i'^k = s_k \frac{\mathbf{r}_y^{\mathbf{k}} \mathbf{p}_\mathbf{i} + T_Y^k}{1 + \eta^k \mathbf{r}_z^{\mathbf{k}} \mathbf{p}_\mathbf{i}}$$

Model Fitting

$$x_i'^k = s_k \frac{\mathbf{r}_x^{\mathbf{k}} \mathbf{p}_\mathbf{i} + T_X^k}{1 + \eta^k \mathbf{r}_z^{\mathbf{k}} \mathbf{p}_\mathbf{i}}$$

$$y_i'^k = s_k \frac{\mathbf{r}_y^{\mathbf{k}} \mathbf{p}_\mathbf{i} + T_Y^k}{1 + \eta^k \mathbf{r}_z^{\mathbf{k}} \mathbf{p}_\mathbf{i}} \quad w_i^k = (1 + \eta^k (\mathbf{r}_z^k . \mathbf{p}_i))^{-1}$$

$$w_i^k (x_i'^k + x_i'^k \eta^k (\mathbf{r}_\mathbf{z}^{\mathbf{k}} . \mathbf{p}_\mathbf{i}) - s^k (\mathbf{r}_X^{\mathbf{k}} . \mathbf{p}_\mathbf{i} + T_X^k)) = 0$$

$$w_i^k (y_i'^k + y_i'^k \eta^k (\mathbf{r}_\mathbf{z}^{\mathbf{k}} . \mathbf{p}_\mathbf{i}) - s^k (\mathbf{r}_Y^{\mathbf{k}} . \mathbf{p}_\mathbf{i} + T_Y^k)) = 0$$

Model Fitting

- Solve for unknowns in five steps:

$$s^k ; \mathbf{p}_i ; \mathbf{R}^k ; T_X^k , T_Y^k ; \square^k$$

- Use linear least squares fit.

- When solving for an unknown, assume other parameters are known.

Least Squares Fit

$$a_j . x \square b_j = 0$$

$$\square_j (a_j . x \square b_j)^2$$

$$\square_j (a_j a_j^T) x = \square_j b_j a_j$$

$$w_i^k (x_i^k \square + x_i^k \square \square^k (\mathbf{r_z^k} . \mathbf{p_i}) \square s^k (\mathbf{r}_X^\mathbf{k} . \mathbf{p_i} + T_X^k)) = 0$$

$$w_i^k (y_i^k \square + y_i^k \square \square^k (\mathbf{r_z^k} . \mathbf{p_i}) \square s^k (\mathbf{r}_Y^\mathbf{k} . \mathbf{p_i} + T_Y^k)) = 0$$

Update for p

$$a_{2k+0} = w_i^k (x_i^k \square^k r_z^k \square s^k r_x^k) \qquad \mathrm{b}_{2\mathrm{k}+0} = w_i^k (s^k T_x^k \square x_i^k)$$

$$a_{2k+1} = w_i^k (y_i^k \square^k r_z^k \square s^k r_y^k) \qquad \mathrm{b}_{2\mathrm{k}+1} = w_i^k (s^k T_y^k \square y_i^k)$$

$$a_j.x \square b_j = 0$$

$$\sum_j (a_j.x \square b_j)^2 \qquad w_i^k (x_i^{\square k} + x_i^{\square k} \square^k (\mathbf{r_z^k}.\mathbf{p_i}) \square s^k (\mathbf{r_X^k}.\mathbf{p_i} + T_X^k)) = 0$$

$$w_i^k (y_i^{\square k} + y_i^{\square k} \square^k (\mathbf{r_z^k}.\mathbf{p_i}) \square s^k (\mathbf{r_Y^k}.\mathbf{p_i} + T_Y^k)) = 0$$

$$\sum_j (a_j a_j^T) x = \sum_j b_j a_j$$

Update for s^k

$$a_{2k+0} = w_i^k (r_x^k.p_i + t_x^k) \qquad \mathrm{b}_{2k+0} = w_i^k (x_i^k + x_i^k \square^k (r_z^k.p_i))$$

$$a_{2k+1} = w_i^k (r_y^k.p_i + t_y^k) \qquad \mathrm{b}_{2k+1} = w_i^k (y_i^k + y_i^k \square^k (r_z^k.p_i))$$

$$y_i \square + y_i \square \square (\mathbf{r}_z.\mathbf{p}_i) \square s(\mathbf{r}_y.\mathbf{p}_i + T_y) = 0$$

Solving for T_x and T_y

$$sT_x = x_i \square + x_i \square \square (\mathbf{r}_z.\mathbf{p}_i) \square s(\mathbf{r}_x.\mathbf{p}_i)$$

$$a_0 = s, b_0 = x_i \square + x_i \square \square (\mathbf{r}_z.\mathbf{p}_i) \square s(\mathbf{r}_x.\mathbf{p}_i)$$

$$sT_y = y_i \square + y_i \square \square (\mathbf{r}_z.\mathbf{p}_i) \square s(\mathbf{r}_y.\mathbf{p}_i)$$

$$a_0 = s, b_0 = y_i \square + y_i \square \square (\mathbf{r}_z.\mathbf{p}_i) \square s(\mathbf{r}_y.\mathbf{p}_i)$$

$$y_i' + y_i'\,\square(\mathbf{r}_z.\mathbf{p}_i) - s(\mathbf{r}_y.\mathbf{p}_i + T_y) = 0$$

Solving for □

$$a_0 = x_i'(\mathbf{r}_z.\mathbf{p}_i), b_0 = s(\mathbf{r}_x.\mathbf{p}_i + T_x) - x_i'$$
$$a_1 = y_i'(\mathbf{r}_z.\mathbf{p}_i), b_1 = s(\mathbf{r}_y.\mathbf{p}_i + T_y) - y_i'$$

Rotation Around an Arbitrary Axis (Rodriguez's Formula)

$$V = (V.n)n + (V - (V.n)n)$$

HW 4.1

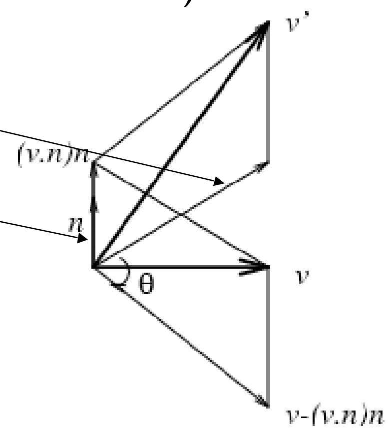
$$V_\perp' = \cos\theta(V - (V.n)n) + \sin\theta(n \times (V - (V.n)n))$$

$$V_\parallel' = (V.n)n$$

$$V' = V_\perp' + V_\parallel'$$

$$V' = \cos\theta\, V + \sin\theta\, n \times V + (1 - \cos\theta)(V.n)n$$

$$V' = V + \sin\theta\, n \times V + (1 - \cos\theta) n \times (n \times V)$$



$$n \times (n \times V) = (V.n)n - V$$

Rotation Around an Arbitrary Axis (Rodriguez's Formula)

$$V' = V + \sin\theta\, n \times V + (1 - \cos\theta)\, n \times (n \times V)$$

$$V' = R(n,\theta)V$$

$$R(n,\theta) = I + \sin\theta\, X(n) + (1 - \cos\theta) X^2(n) \quad \text{HW 4.2}$$

$$X(n) = \begin{bmatrix} 0 & -n_z & n_x \\ n_z & 0 & -n_x \\ -n_y & n_x & 0 \end{bmatrix}$$

Rotation Around an Arbitrary Axis (Rodriguez's Formula)

$$r = \|r\| \frac{r}{\|r\|} = \theta n$$

$$R(n,\theta) = I + \sin\theta\, X(n) + (1 - \cos\theta) X^2(n)$$

$$X(n) = \begin{bmatrix} 0 & -n_z & n_x \\ n_z & 0 & -n_x \\ -n_y & n_x & 0 \end{bmatrix}$$

$$R(r,\theta) = I + \sin\theta \frac{X(r)}{\|r\|} + (1 - \cos\theta) \frac{X^2(r)}{\|r\|^2}$$

$$X(r) = \begin{bmatrix} 0 & -r_z & r_x \\ r_z & 0 & -r_x \\ -r_y & r_x & 0 \end{bmatrix}$$

$$R^{it+1} \; \square \; \widetilde{R} R^{it}$$

$$R(m) = I + \sin\square \frac{X(m)}{\square} + \frac{X^2(m)}{\square^2}(1 \square \cos\square)$$

$$\widetilde{R} \; \square \; I + X(m)$$

$$m = \square n = (m_x, m_y, m_z)$$

$$\widetilde{r}_x^{\,k} = (1, \square m_z, m_y)$$

$$\widetilde{r}_y^{\,k} = (m_z, 1, \square m_x)$$

$$\widetilde{r}_z^{\,k} = (\square m_y, m_x, 1)$$

$$w_i^k (x\square + x\square\square^k (\mathbf{r_z^k} . \mathbf{p_i}) \square s^k (\mathbf{r}_X^{\mathbf{k}} . \mathbf{p_i} + T_X^k)) = 0$$

$$w_i^k (y\square + y\square\square^k (\mathbf{r_z^k} . \mathbf{p_i}) \square s^k (\mathbf{r}_Y^{\mathbf{k}} . \mathbf{p_i} + T_Y^k)) = 0$$

$$\widetilde{r}_x^{\,k} = (1, \square m_z, m_y)$$

$$\widetilde{r}_y^{\,k} = (m_z, 1, \square m_x)$$

$$\widetilde{r}_z^{\,k} = (\square m_y, m_x, 1)$$

$$R^{it+1} \; \square \; \widetilde{R} R^{it}$$

$$w_i^k (x_i^k + x_i^k \square (\widetilde{r}_z^{\,k} . q_i) \square s^k (\widetilde{r}_x^{\,k} . q_i + t_x^k)) = 0$$

$$w_i^k (y_i^k + y_i^k (\widetilde{r}_z^{\,k} . q_i) \square s^k (\widetilde{r}_y^{\,k} . q_i + t_y^k)) = 0$$

Solving for rotation:

$$R^{it+1} \square \tilde{R} R^{it}$$

$$q_i = R^{it} p_i$$

$$\tilde{r}_z^k = (\square m_y, m_x, 1)$$

$$\tilde{r}_x^k = (1, \square m_z, m_y)$$

$$\tilde{r}_y^k = (m_z, 1, \square m_x)$$

$$w_i^k (x\square + x\square\square^k (\mathbf{r_z^k}.\mathbf{p_i}) \square s^k (\mathbf{r_X^k}.\mathbf{p_i} + T_X^k)) = 0$$

$$w_i^k (y\square + y\square\square^k (\mathbf{r_z^k}.\mathbf{p_i}) \square s^k (\mathbf{r_Y^k}.\mathbf{p_i} + T_Y^k)) = 0$$

$$\Downarrow$$

$$w_i^k (x\square + x\square\square(\tilde{r}_z^k.q_i) \square s^k (\tilde{r}_x^k.q_i + T_x^k)) = 0$$

$$w_i^k (y\square + y\square\square(\tilde{r}_z^k.q_i) \square s^k (\tilde{r}_y^k.q_i + T_y^k)) = 0$$

$$\Downarrow$$

$$x\square + x\square\square(\square m_y q_x + m_x q_y + q_z) \square s(q_x \square m_z q_y + m_y q_z + T_x) = 0$$

$$y\square + y\square\square(\square m_y q_x + m_x q_y + q_z) \square s(m_z q_x + q_y \square m_x q_z + T_y) = 0$$

$$y\square + y\square\square(\square m_y q_x + m_x q_y + q_z) \square s(m_z q_x + q_y \square m_x q_z + T_y) = 0$$

$$\begin{bmatrix} \square x\square\square q_y & \square x\square\square q_x \square s q_z & s q_y \end{bmatrix} \begin{bmatrix} \square m_x \\ \square m_y \\ \square m_z \end{bmatrix} = \begin{bmatrix} \square s T_x \square x\square\square x\square\square q_z + s q_x \end{bmatrix}$$

$$\begin{bmatrix} \square y\square\square q_y + s q_z & \square y\square\square q_x & \square s q_x \end{bmatrix} \begin{bmatrix} \square m_x \\ \square m_y \\ \square m_z \end{bmatrix} = \begin{bmatrix} \square s T_y \square y\square\square y\square\square q_z + s q_y \end{bmatrix}$$

$$a_0 = \begin{bmatrix} \square x\square\square q_y & \square x\square\square q_x + s q_z & s q_y \end{bmatrix} \quad b_0 = \begin{bmatrix} \square s T_x \square x\square\square x\square\square q_z + s q_x \end{bmatrix}$$

$$a_1 = \begin{bmatrix} \square y\square\square q_y \square s q_z & \square y\square\square q_x & s q_x \end{bmatrix} \quad b_1 = \begin{bmatrix} \square s T_y \square y\square\square y\square\square q_z + s q_y \end{bmatrix}$$