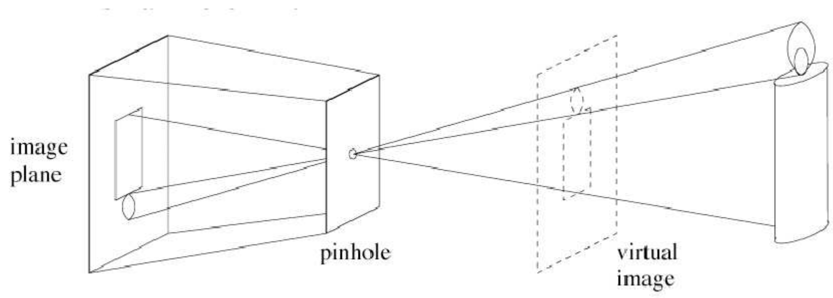
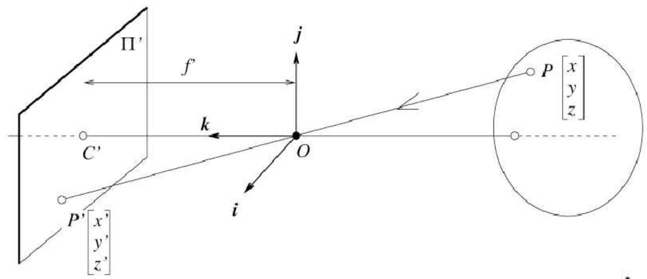
Multi-View Geometry

Review: Pinhole Camera



Review: Perspective Projection



$$x' = f' \frac{x}{z}$$

$$y' = f' \frac{y}{z}$$

Review: Perspective Projection

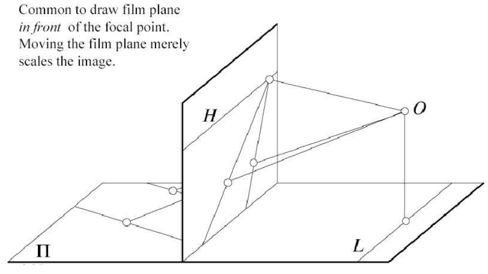
Points go to Points

Lines go to Lines

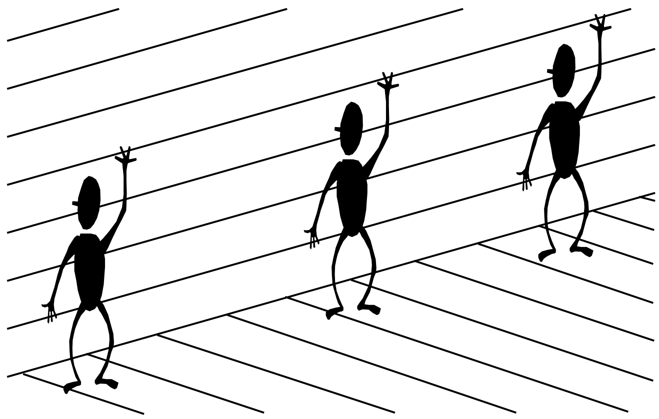
Planes go to whole image or Half-planes

Polygons go to Polygons

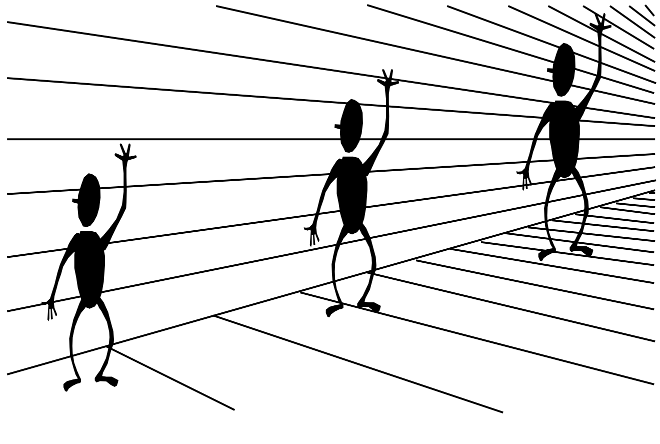
Common to draw film plane *in front* of the focal point. Moving the film plane merely scales the image.



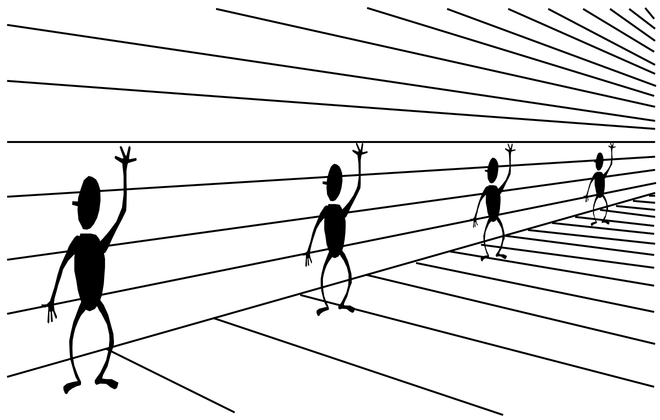
Perspective cues



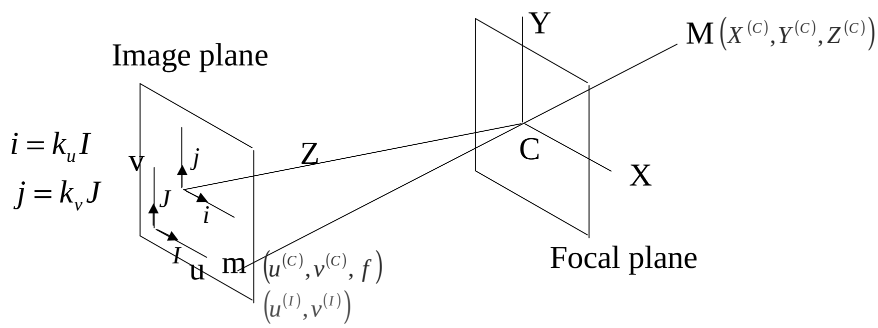
Perspective cues



Perspective cues



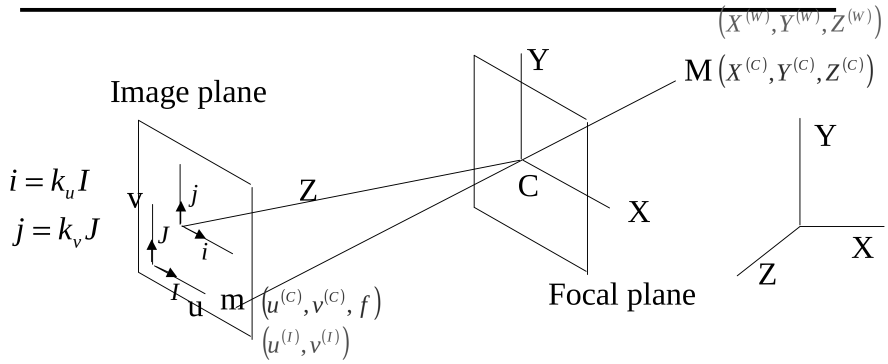
Review: Intrinsic Camera Parameters



$$\begin{bmatrix} U^{(new)} \\ V^{(new)} \\ S \end{bmatrix} = \begin{bmatrix} -f_u & 0 & u_0 & 0 \\ 0 & -f_v & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X^{(C)} \\ Y^{(C)} \\ Z^{(C)} \\ 1 \end{bmatrix}$$

$$f_u = fk_u$$
$$f_v = fk_v$$

Review: Extrinsic Parameters



By Rigid Body Transformation:

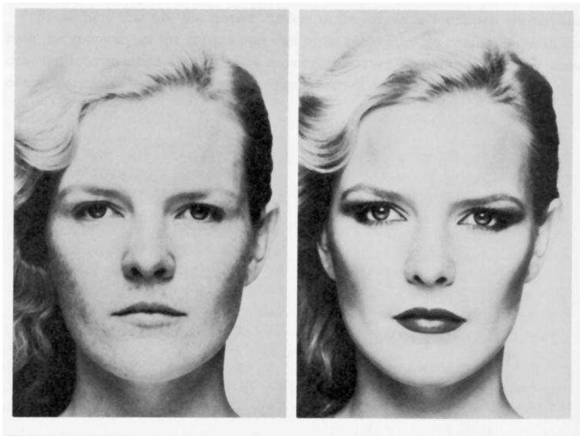
$$\begin{bmatrix} X^{(C)} \\ Y^{(C)} \\ Z^{(C)} \\ 1 \end{bmatrix} = \begin{bmatrix} R_{3\times3} & T_{3\times1} \\ 0_{1\times3} & 1 \end{bmatrix} \begin{bmatrix} X^{(W)} \\ Y^{(W)} \\ Z^{(W)} \\ 1 \end{bmatrix} \Rightarrow M^{(C)} = DM^{(W)}$$

Recovering 3D from images

What cues in the image provide 3D information?

Visual cues

Shading

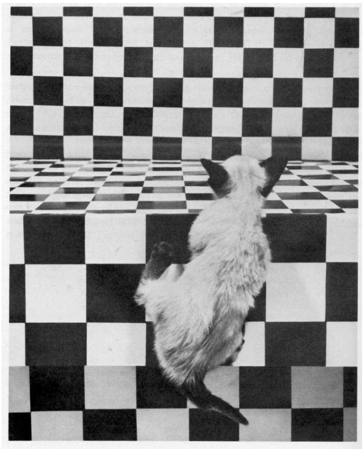


Merle Norman Cosmetics, Los Angeles

Visual cues

Shading

Texture



***The Visual Cliff*, by William Vandivert, 1960**

Visual cues

Shading

Texture

Focus

From *The Art of Photography*, Canon

Visual cues

Shading

Texture

Focus

Motion

Visual cues

Shading

Texture

Focus

Motion

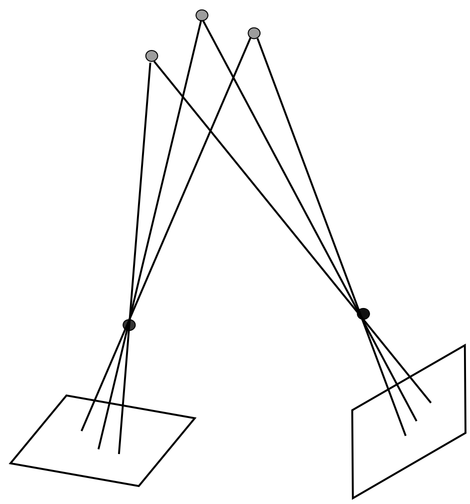
Shape From X

- X = shading, texture, focus, motion, ...

Multi-View Geometry

Relates

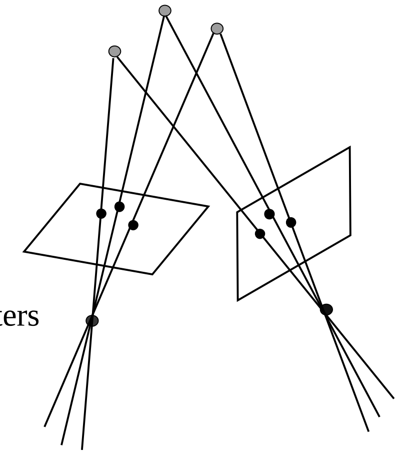
- 3D World Points
- Camera Centers
- Camera Orientations



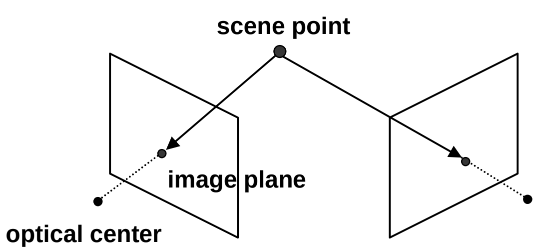
Multi-View Geometry

Relates

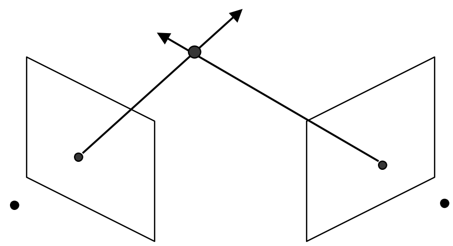
- 3D World Points
- Camera Centers
- Camera Orientations
- Camera Intrinsic Parameters
- Image Points



Stereo



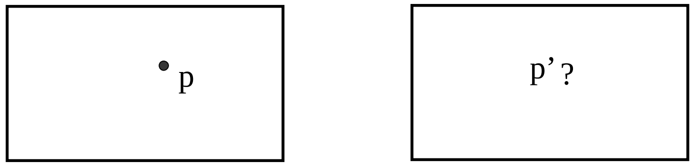
Stereo



Basic Principle: Triangulation

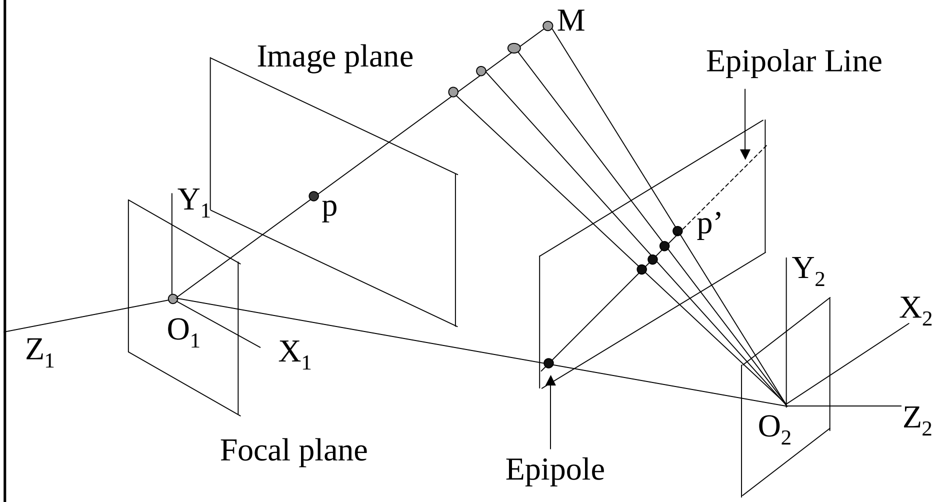
- Gives reconstruction as intersection of two rays
- Requires
  - calibration
  - ***point correspondence***

Stereo Constraints



Given p in left image, where can the corresponding point p' in right image be?

Stereo Constraints



Epipolar Constraint

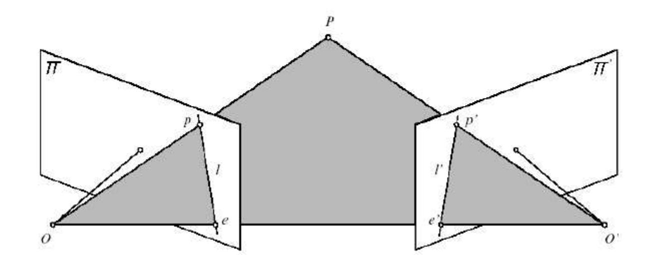


FIGURE 11.1: Epipolar geometry: the point P, the optical centers O and O' of the two cameras, and the two images p and p' of P all lie in the same plane.

All epipolar lines contain epipole, the image of other camera center.

From Geometry to Algebra

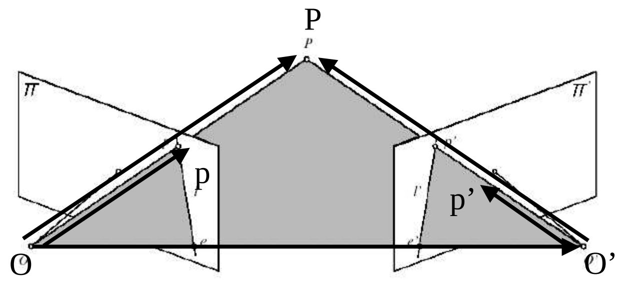
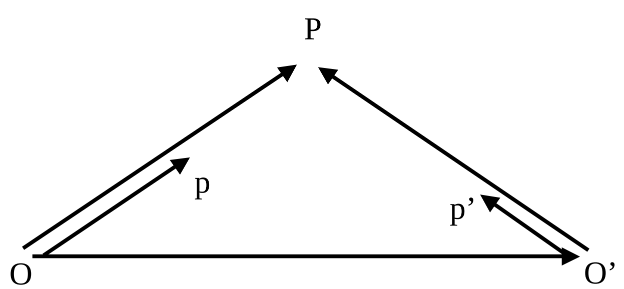


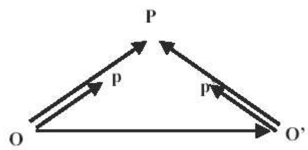
FIGURE 11.1: Epipolar geometry: the point P, the optical centers O and O' of the two cameras, and the two images p and p' of P all lie in the same plane.

From Geometry to Algebra

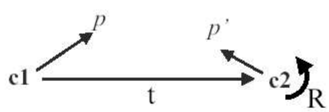


The epipolar constraint: these vectors are coplanar:

$$\overrightarrow{Op} \cdot [\overrightarrow{OO'} \times \overrightarrow{O'p'}] = 0$$



$$\overrightarrow{Op} \cdot [\overrightarrow{OO'} \times \overrightarrow{O'p'}] = 0$$



p,p' are image coordinates of P in c1 and c2…

c2 is related to c1 by rotation R and translation t

$$\boldsymbol{p} \cdot \left[\boldsymbol{t} \times (\mathcal{R}\boldsymbol{p}')\right] = 0$$

Linear Constraint:
Should be able to express as matrix multiplication.

Review: Matrix Form of Cross Product

The vector cross product also acts on two vectors and returns a third vector. Geometrically, this new vector is constructed such that its projection onto either of the two input vectors is zero.

$$\vec{a} \times \vec{b} = \begin{bmatrix} a_y b_z - a_z b_y \\ a_z b_x - a_x b_z \\ a_x b_y - a_y b_x \end{bmatrix}$$

$$\vec{a} \times \vec{b} = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} = \vec{c} \qquad \begin{matrix} \vec{a} \cdot \vec{c} = 0 \\ \vec{b} \cdot \vec{c} = 0 \end{matrix}$$

Review: Matrix Form of Cross Product

$$\vec{a} \times \vec{b} = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} = \vec{c}$$

$$\vec{a} \cdot \vec{c} = 0$$

$$\vec{b} \cdot \vec{c} = 0$$

$$[a_x] = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}$$

$$\boxed{\vec{a} \times \vec{b} = [a_x]\vec{b}}$$

Matrix Form

$$\boldsymbol{p} \cdot [\boldsymbol{t} \times (\mathcal{R}\boldsymbol{p}')] = 0$$

$$\vec{a} \times \vec{b} = [a_x]\vec{b}$$

$$p^T [t_x] \Re p' = 0$$

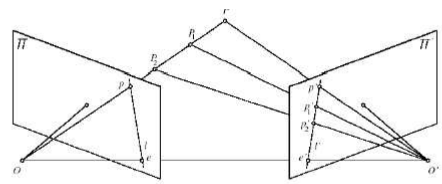
$$\varepsilon = [t_x] \Re$$

$$\boxed{\boldsymbol{p}^T \mathcal{E} \boldsymbol{p}' = 0}$$

The Essential Matrix

Matrix that relates image of point in one camera to a second camera, given translation and rotation.

$$\varepsilon = [t_x]\Re$$



$$\boldsymbol{p}^T \mathcal{E} \boldsymbol{p}' = 0$$

$$\vec{a} \times \vec{b} = [a_x]\vec{b}$$