Change Detection

C. Stauffer and W.E.L. Grimson, “Learning patterns of activity using real time tracking,” IEEE Trans. On PAMI, 22(8):747-757, Aug 2000

Motivation

•Detection of interesting objects in videos is the first step in the process of automated surveillance and tracking.

•Focus of attention method greatly reduces the processing-time required for tracking and activity recognition.

Introduction

Objectives:

- Given a sequence of images from a stationary camera identify pixels comprising 'moving' objects.
- We call the pixels comprising 'moving' objects as 'foreground pixels' and the rest as 'background pixels'

General Solution

–Model properties of the scene (e.g. color, texture e.t.c) at each pixel.

–Significant change in the properties indicates an interesting change.

Introduction

Problems in Realistic situations:

- Moving but uninteresting objects
  - e.g. trees, flags or grass.
- Long term illumination changes
  - e.g. time of day.
- Quick illumination changes
  - e.g. cloudy weather
- Shadows
- Other Physical changes in the background
  - e.g. dropping or picking up of objects
- Initialization

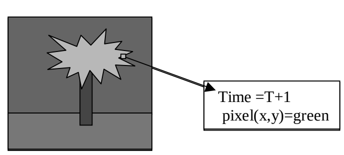
Issues

- Adaptivity
  - Background model must be adaptive to changes in background.
- Multiple Models
  - Multiple processes generate color at every pixel. The background model should be able to account for these processes.
- Weighting the observations (models)
  - The system must be able to weight the observation to make decisions. For example, the observations made a long time back should have less weight than the recent observations. Similarly, the frequent observations are more important than the ones with less occurrence.

Color based Background Modeling

Pixel level Color Modeling

- Multiple Processes are generating color '*x*' at each pixel
  - Where $x=[R,G,B]^T$

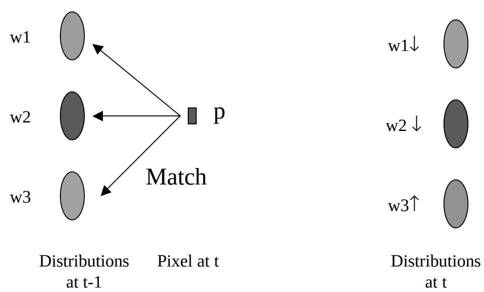


Color based Background Modeling

At each frame

For each pixel

- Calculate distance of pixel's color value from each of the associated K Guassian distributions



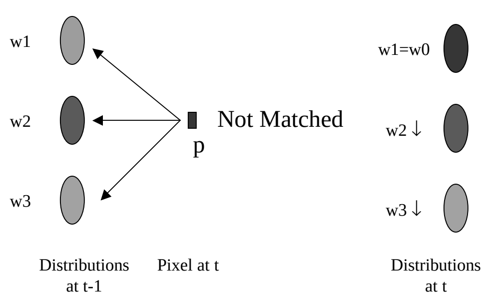
p is background pixel If w3 > Threshold
p is foreground pixel otherwise

Color based Background Modeling

At each frame

For each pixel

- Calculate distance of pixel's color value from each of the associated K Guassian distributions



p is a foreground pixel

$w1 \leq w2 \leq w3$

Color based Background Modeling

For each pixel *(i,j)* at time '*t*' each process is modeled as a Gaussian distribution.

– Guassian distribution is described by a mean '*m*' and a covariance matrix **Σ**.

$$N(x_{i,j}^t \mid m_{i,j}^t, \Sigma_{i,j}^t) = \frac{1}{(2\pi)^{\frac{n}{2}} \mid \Sigma_{i,j}^t \mid} e^{-\frac{1}{2}(x_{i,j}^t - m_{i,j}^t)^T (\Sigma_{i,j}^t)^{-1} (x_{i,j}^t - m_{i,j}^t)}$$

$x_{i,j}^t$ is 3x1 vector (RGB value) at pixel (i,j) at time t

$m_{i,j}^t$ is 3x1 mean vector of Gaussian at pixel (i,j) at time t

$\Sigma_{i,j}^t$ is 3x3 covariance matrix at pixel (i,j) at time t

- Each Pixel is modeled as a mixture of Gaussians.
  - –Weight associated with each distribution signifying relevance in recent time.

Mean, Variance and Covariance

Let two features x and y and n observations of each feature be $x_1, x_2, \ldots, x_n$ and $y_1, y_2, \ldots, y_n$ respectively.

Mean: $$m = \begin{bmatrix} m_x & m_y \end{bmatrix}^T = \frac{1}{n}\begin{bmatrix} \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} y_i \end{bmatrix}^T$$

Variance: $$\sigma_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - m_x)^2 \quad \sigma_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - m_y)^2$$

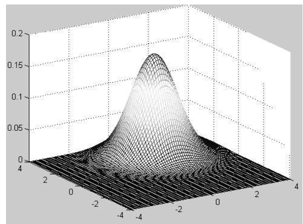
Covariance: $$\sigma_{xy}^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - m_x)(y_i - m_y)$$

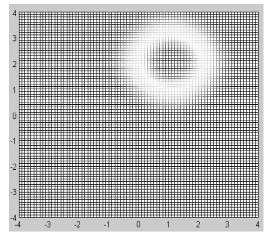
Covariance Matrix: $$\Sigma = \begin{bmatrix} \sigma_x^2 & \sigma_{xy}^2 \\ \sigma_{xy}^2 & \sigma_y^2 \end{bmatrix}$$

2D Gaussian

$$m = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

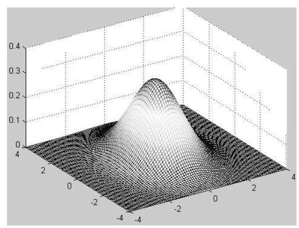
$$\Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

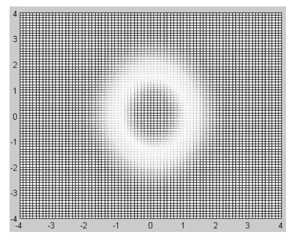




$$m = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

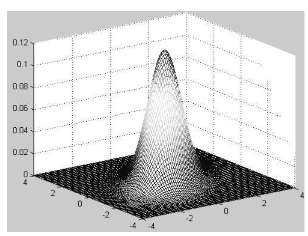


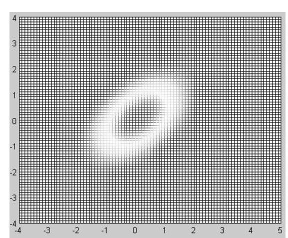


2D Gaussian

$$m = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

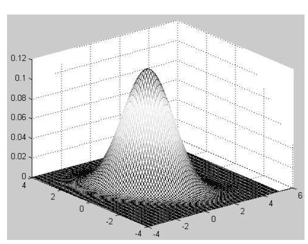
$$\Sigma = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$$

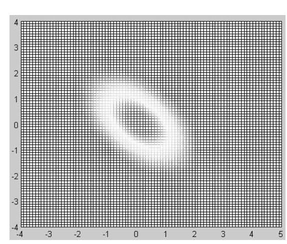




$$m = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Sigma = \begin{bmatrix} 1 & -0.5 \\ -0.5 & 1 \end{bmatrix}$$





Mahalanobis Distance

Given a vector x, and a normal distribution $N(m,\Sigma)$, the Mahalanobis distance from feature vector x to the sample mean m is given by

$$d = \sqrt{(x-m)^T (\Sigma)^{-1} (x-m)}$$

Parameter Update

Let $x_1, x_2, \ldots, x_n$ be the n observations and m_n and σ_n^2 be the mean and variance of these observations respectively. Let x_{n+1} be a new observation, then the updated mean and variance are given by

$$m_{n+1} = \frac{1}{n+1}\sum_{i=1}^{n+1} x_i = m_n + \frac{1}{n+1}\left(x_{n+1} - m_n\right)$$

$$\sigma_{n+1}^2 = \frac{1}{n}\sum_{i=1}^{n+1}\left(x_i - m_{n+1}\right)^2 = \frac{n-1}{n}\sigma_n^2 + \frac{1}{n+1}\left(x_{n+1} - m_n\right)^2$$

Assignment
Due April 15, 2003

Parameter Update

- If a match is found with the k^{th} Gaussian, update parameters

$$m_{i,j}^{t,k} = (1-\rho) m_{i,j}^{t-1,k} + \rho x_{i,j}^{t}$$

$$\Sigma_{i,j}^{t,k} = (1-\rho)\Sigma_{i,j}^{t-1,k} + \rho (x_{i,j}^{t} - m_{i,j}^{t})(x_{i,j}^{t} - m_{i,j}^{t})^{T}$$

•where *p* is a learning parameter

Color based Background Modeling

- If a match is not found
  - Replace lowest weight distribution with a new distribution such that

$$m_{i,j}^{t,new} = x_{i,j}^{t}$$

$$\Sigma_{i,j}^{t,new} = \Sigma^{initial}$$

- The prior weights of K distributions are adjusted as

$$\omega_{i,j}^{t-1} = (1-\alpha)\omega_{i,j}^{t-1} + \alpha (M_{i,j}^{t-1})$$

• M is1 for model that matched and 0 for others

Color based Background Modeling

- Foreground= Matched distributions with weight< T
  + Unmatched pixels

Summary

- Each pixel is an independent statistical process, which may be combination of several processes.
  - Swaying branches of tree result in a bimodal behavior of pixel intensity.
- The intensity is fit with a mixture of K Gaussians.

$$N(x_{i,j}^t \mid m_{i,j}^t, \Sigma_{i,j}^t) = \frac{1}{(2\pi)^{\frac{n}{2}} |\Sigma_{i,j}^t|} e^{-\frac{1}{2}(x_{i,j}^t - m_{i,j}^t)^T (\Sigma_{i,j}^t)^{-1} (x_{i,j}^t - m_{i,j}^t)}$$

- For simplicity, it may be assumed that RGB color channels are independent and have the same variance σ^2. In this case $\Sigma_{i,j}^t = \sigma^2 I$, where I is a 3x3 unit matrix.

Summary

- Every new pixel is checked against all existing distributions. The match is the distribution with Mahalanobis distance less than a threshold.

- The mean and variance of unmatched distributions remain unchanged. For the matched distributions they are updated as

$$m_{i,j}^{t,k} = (1-\rho) m_{i,j}^{t-1,k} + \rho x_{i,j}^{t}$$

$$\Sigma_{i,j}^{t,k} = (1-\rho)\Sigma_{i,j}^{t-1,k} + \rho (x_{i,j}^{t} - m_{i,j}^{t})(x_{i,j}^{t} - m_{i,j}^{t})^{T}$$

Summary

- For the unmatched pixel, replace the lowest weight Gaussian with the new Gaussian with mean at the new pixel and an initial estimate of covariance matrix.

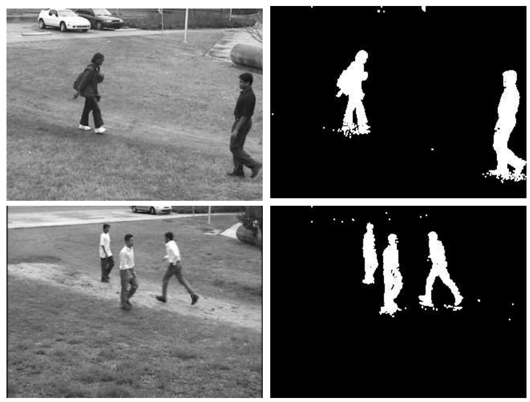
- The weights are adjusted:

$$\omega_{i,j}^{t-1} = (1-\alpha)\omega_{i,j}^{t-1} + \alpha (M_{i,j}^{t-1})$$

$$M_{ij}^{t-1} = \begin{cases} 1 & \text{if distribution matches} \\ 0 & \text{otherwise} \end{cases}$$

- Foreground= Matched distributions with weight< T + Unmatched pixels

Results



Color based Background Modeling

Pros

–Handles slow changes in illumination conditions

–Can accommodate physical changes in the background after a certain time interval.

–Initialization with moving objects will correct itself after a certain time interval.

Color based Background Modeling

Cons

– Cannot handle quick changes in illumination conditions e.g. cloudy weather

– Initialization with moving objects

–Shadows

–Physical Changes in Background

Implementation Issues in Programming Assignment #4

Estimation of Global Flow

Iterative

Initial Estimate $\mathbf{a} = [a_1 \quad a_2 \quad b_1 \quad a_3 \quad a_4 \quad b_2]^T$

Image 't'

Image 't+1'

Warp by **a**

Compute **A** and **B**

Solve $\mathbf{A}\delta\mathbf{a} = \mathbf{B}$

Normalization

0

M

0 N

Normalization

0

1

0 1

$$x^{new} = \frac{x}{N}$$

$$y^{new} = \frac{y}{M}$$

$$I_x = \lim_{\Delta x \to 0} \frac{I(x+\Delta x, y) - I(x, y)}{\Delta x}$$

$$\Delta x = \frac{1}{N}$$

$$I_x = N\left(\frac{\partial}{\partial x} G_\sigma\right) * I$$