Shape Descriptors

Desired Properties

- Uniqueness
- Invariance
  - Size
  - Rotations
  - Translations
  - Noise

Moments

General Moments

$$m_{pq} = \int \int x^p y^q B(x, y) dxdy$$

Discrete Form

$$m_{pq} = \sum_x \sum_y x^p y^q B(x, y)$$

Uniqueness Theorem

- The double moment sequence $\{m_{pq}\}$ is uniquely determined by B(x,y) and conversely B(x,y) is uniquely determined by $\{m_{pq}\}$

Characteristic Function/ Moment Generating Function

Characteristic Function:

$$\phi(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp(iux + ivy) B(x,y)\,dxdy$$

Moment Generating Function:

$$M(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp(ux + vy) B(x,y)\,dxdy$$

Characteristic Function/ Moment Generating Function

Characteristic Function:

$$\phi(u,v) = \sum_{p=0}^{\infty} \sum_{q=0}^{\infty} m_{pq} \frac{(iu)^p}{p!} \frac{(iv)^q}{q!}$$

Moment Generating Function:

$$M(u,v) = \sum_{p=0}^{\infty} \sum_{q=0}^{\infty} m_{pq} \frac{u^p}{p!} \frac{v^q}{q!}$$

Central Moments

$$\mu_{pq} = \int\int (x-\bar{x})^p (y-\bar{y})^q B(x,y)\ d(x-\bar{x})d(y-\bar{y})$$

$$\bar{x} = \frac{m_{10}}{m_{00}},\quad \bar{y} = \frac{m_{01}}{m_{00}}$$ Centroid

Translation Invariant

Central Moments

$$\mu_{00} = m_{00} \equiv \mu$$
$$\mu_{01} = 0$$
$$\mu_{10} = 0$$
$$\mu_{20} = m_{20} - \mu\bar{x}^2$$
$$\mu_{11} = m_{11} - \mu\overline{xy}$$
$$\mu_{02} = m_{02} - \mu\bar{y}^2$$
$$\mu_{30} = m_{30} - 3m_{20}\bar{x} + 2\mu\bar{x}^3$$
$$\mu_{21} = m_{21} - m_{20}\bar{y} - 2m_{11}\bar{x} + 2\mu\bar{x}^2 y$$
$$\mu_{12} = m_{12} - m_{02}\bar{x} - 2m_{11}\bar{y} + 2\mu\bar{x}y^2$$
$$\mu_{03} = m_{03} - 3m_{02}\bar{y} + 2\mu\bar{y}^3$$

Hu Moments

$$v_1 = u_{20} + u_{02}$$

$$v_2 = \left(u_{20} - u_{02}\right)^2 + 4u_{11}^2$$

$$v_3 = \left(u_{30} - 3u_{12}\right)^2 + \left(3u_{12} - u_{03}\right)^2$$

$$v_4 = \left(u_{30} + u_{12}\right)^2 + \left(u_{21} + u_{03}\right)^2$$

Translation, Rotation, Scaling Invariant

Hu Moments

$$v_5 = (u_{30} - 3u_{12})(u_{30} + u_{12})\left[(u_{30} + u_{12})^2 - 3(u_{21} + u_{03})^2\right]$$
$$+ (3u_{21} - u_{03})(u_{21} + u_{03})\left[3(u_{30} + u_{12})^2 - (u_{21} + u_{03})^2\right]$$

$$v_6 = (u_{20} - u_{02})\left[(u_{30} + u_{12})^2 - (u_{21} - u_{03})^2\right.$$
$$\left. + 4u_{11}(u_{30} + u_{12})(u_{21} + u_{03})\right]$$

$$v_7 = (3u_{21} - u_{03})(u_{30} + u_{12})\left[(u_{30} + u_{12})^2 - 3(u_{30} + u_{12})^2\right]$$
$$+ (u_{30} - 3u_{12})(u_{21} + u_{03})\left[3(u_{30} + u_{12})^2 - (u_{21} + u_{03})^2\right]$$

Hu Moments

Table 8.2 Moment Invariants for the Images in Figs. 8.24(a)–(e)

Invariant (Log)	*Original*	*Half Size*	*Mirrored*	*Rotated 2°*	*Rotated* [illegible]
ϕ_1	6.249	6.226	6.919	6.253	6.318
ϕ_2	17.180	16.954	19.955	17.270	16.803
ϕ_3	22.655	23.531	26.689	22.836	19.724
ϕ_4	22.919	24.236	26.901	23.130	20.437
ϕ_5	45.749	48.349	53.724	46.136	40.525
ϕ_6	31.830	32.916	37.134	32.068	29.315
ϕ_7	45.589	48.343	53.590	46.017	40.470

Hu moments

Medial Axis Transform

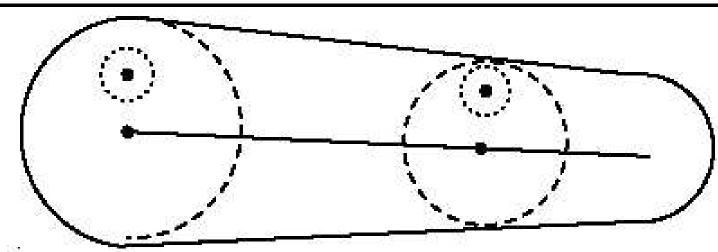

Figure 4.19: Medial axis transform. The points on the medial axis are the centers of the maximal circular neighborhoods totally contained in the shape. Note that the centers of the smaller circles (shown dotted) do not constitute the medial axis fo this shape.

Medial Axis Transform

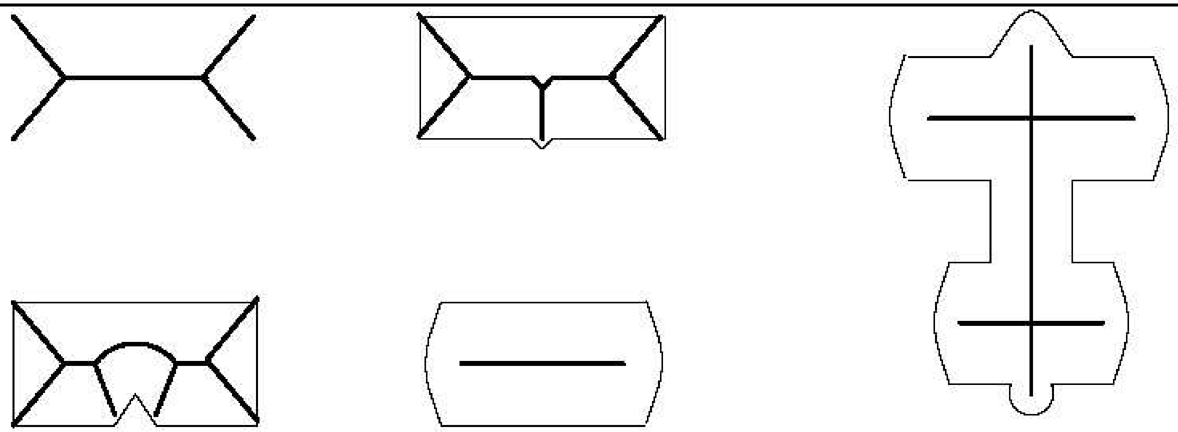

Figure 4.24: Some examples of medial axis transform.

Medial Axis Transform

1. Iteratively compute f^k as follows:

$$f^k(x,y) = f^0(x,y) + \min(f^{k-1}(p,q))$$

$\forall (p,q)$ such that $distance((x,y),(p,q)) \leq 1$.

2. Medial axis is given by all points (x,y) such that:

$$f^k(x,y) \geq f^k(p,q),$$

$\forall (p,q)$ such that $distance((x,y),(p,q)) \leq 1$.

Figure 4.20: Iterative algorithm for computing medial axis transform.

Medial Axis Transform

1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1

(a)

1	1	1	1	1	1
1	2	2	2	2	1
1	2	2	2	2	1
1	2	2	2	2	1
1	1	1	1	1	1

(b)

1	1	1	1	1	**1**
1	**2**	2	2	**2**	1
1	2	**3**	**3**	2	1
1	**2**	2	2	**2**	1
1	1	1	1	1	**1**

(c)

Figure 4.22: Medial axis transform. (a) Rectangular shape ($f^0(x,y)$). The background pixels, which are not shown, are all '0". (b) intermediate step ($f^1(x,y)$). (c) $f^2(x,y)$. The points in Medial axis shown in boldface.

Medial Axis Transform

1. Iteratively compute g^k as follows:

$$g^k(x,y) = \begin{cases} \max[0, (\max g^{k-1}(p,q)) - 1] & \text{if } g^{k-1} = 0 \\ g^{k-1} & \text{otherwise} \end{cases}$$

$\forall(p,q)$ such that $distance((x,y),(p,q)) \leq 1$.

Figure 4.21: Iterative algorithm for inverse medial axis transform.

Medial Axis Transform

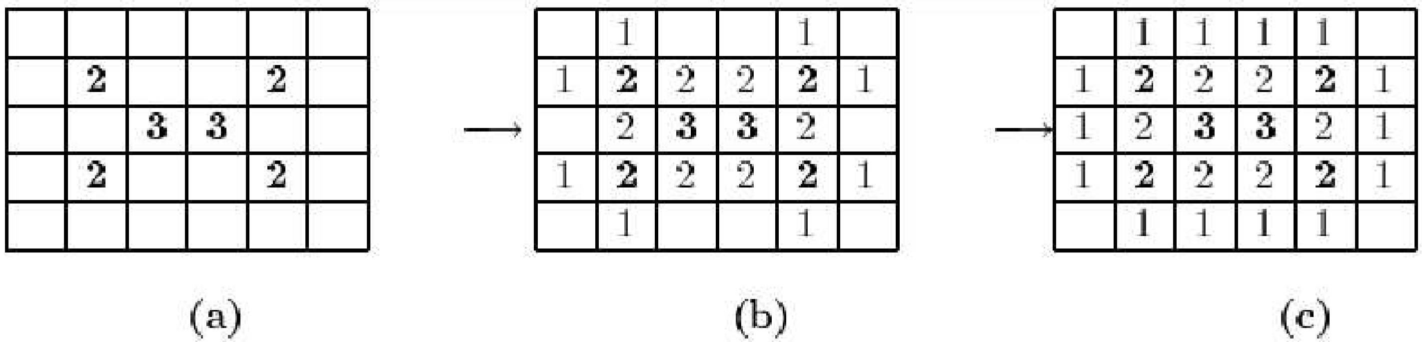

Figure 4.23: Inverse medial axis transform. (a) Medial axis ($g^0(x, y)$). (b) Intermediate step ($g^1(x, y)$). (c) Original shape ($g^2(x, y)$).

Suggested Reading

- M-K. Hu., “Visual pattern recognition by moment invariants,” Computer methods in image analysis.
- H. Blum, “A transformation for extracting new descriptors of shape,” Computer methods in image analysis.
- Chapter 4, Mubarak Shah, “Fundamentals of Computer Vision”.