Deformable Contours
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- Minimize the Energy Functional

\[ E = \int \left[ \alpha(s)E_{\text{cont}} + \beta(s)E_{\text{curv}} + \gamma(s)E_{\text{image}} \right] ds \]

- Where the integral is taken along the contour \( c \) and each of the energy terms in the functional is a function of \( c \) or the derivatives of \( c \) with respect to \( s \). The parameters \( \alpha, \beta, \) and \( \gamma \) control the relative influence of the corresponding energy term, and can vary along \( c \).
Deformable Contours

- Continuity

\[ E_{cont} = \left\| \frac{dc}{ds} \right\|^2 \]

\[ E_{cont} = \left\| p_i - p_{i-1} \right\|^2 \quad \text{Discrete Approximation} \]

\[ E_{cont} = \left( \bar{d} - \left\| p_i - p_{i-1} \right\| \right)^2 \quad \text{A better form} \]
Deformable Contours

- Curvature (Smoothness)

\[ E_{\text{curv}} = \left\| \frac{d^2 c}{ds^2} \right\|^2 \]

\[ E_{\text{cont}} = \left\| p_{i-1} - 2p_i + p_{i+1} \right\|^2 \text{ Discrete Approximation} \]
Deformable Contours

- **Image (Edge Attraction)**

\[ E_{curv} = -\| \Delta I \| \]
Greedy Algorithm (Williams & Shah)

Let \( I \) be the intensity image and \( p_j, \ldots p_k \) be the initial positions of the snake points.

While a fraction greater than \( f \) of the snake points move in an iteration:
1. For each \( i \), find the location of \( N(p_i) \) for which the functional is minimum and move the snake point \( p_i \) to that location.
2. For each \( i \), estimate the curvature \( k \) of the snake and look for local maxima. Set \( \beta(j)=0 \) for all \( p_j \) at which the curvature has a local maximum and is above certain threshold and at which the image gradient is above certain threshold.
3. Update the value of the average distance
Suggested Reading

- Chapter 5, Emanuele Trucco, Alessandro Verri, "Introductory Techniques for 3-D Computer Vision"
Programming Assignment #2

\[ \frac{\partial}{\partial x} G_\sigma = -\frac{x}{2\pi\sigma^4} e^{\frac{x^2 + y^2}{2\sigma^2}} \]

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Programming Assignment #2

\[ \frac{\partial}{\partial y} G_\sigma = -\frac{y}{2\pi\sigma^4} e^{-\frac{x^2+y^2}{2\sigma^2}} \]
Programming Assignment #2

\[ S_x = \left( \frac{\partial}{\partial x} G_\sigma \right) * I \]

\[ S_y = \left( \frac{\partial}{\partial y} G_\sigma \right) * I \]

\[ |\Delta| = \sqrt{S_x^2 + S_y^2} \]
Programming Assignment #2

Least Square Fitting

Let the line be: \( y = ax + b \)

Solve

\[
\begin{bmatrix}
\sum_{i} x_i y_i \\
\sum_{i} y_i
\end{bmatrix}
= 
\begin{bmatrix}
\sum_{i} x_i^2 & \sum_{i} x_i \\
\sum_{i} x_i & n
\end{bmatrix}
\begin{bmatrix}
a \\
b
\end{bmatrix}
\]
Programming Assignment #2

Maximum Likelihood Fitting

Let the line be: \( ax + by + c = 0 \)

Find the Eigen Vectors \(
\begin{bmatrix}
a \\
b
\end{bmatrix}
\)

of the matrix

\[
\begin{bmatrix}
\sum x_i^2 - \left( \frac{\sum x_i}{n} \right)^2 & \sum x_i y_i - \left( \frac{\sum x_i}{n} \right) \left( \frac{\sum y_i}{n} \right) \\
\sum x_i y_i - \left( \frac{\sum x_i}{n} \right) \left( \frac{\sum y_i}{n} \right) & \sum y_i^2 - \left( \frac{\sum y_i}{n} \right)^2
\end{bmatrix}
\]

Compute \( c = -a \frac{\sum x_i}{n} - b \frac{\sum y_i}{n} \)