CAP5415 Computer Vision
Spring 2003

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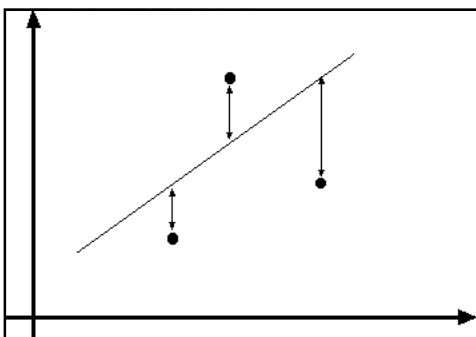
- Imaging Geometry
- Camera Modeling and Calibration
- Filtering and Convolution
- Edge Detection
- Line & Curve Fitting
- Deformable Contours

Least Squares Fit

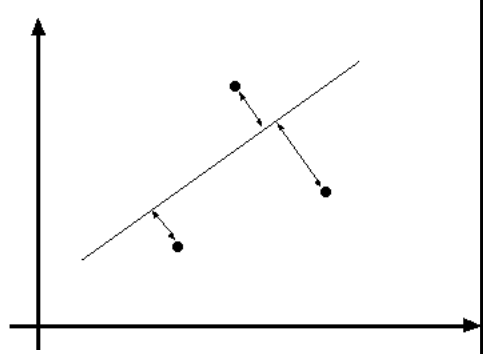
- Standard linear solution to a classical problem.
- Poor Model for vision applications.

$$y = ax + b = f(x, a, b)$$

$$\text{Minimize} \sum_i \left[y_i - f(x_i, a, b) \right]^2$$



Line fitting can be max. likelihood - but choice of model is important



Maximum Likelihood

Maximize the Log likelihood function L

$$L = -\frac{\sum_i (ax_i + by_i + c)^2}{2\sigma^2}$$

Given constraint

$$a^2 + b^2 = 1$$

Who came from which line?

- Assume we know how many lines there are - but which lines are they?
  - easy, if we know who came from which line
- Strategies
  - Incremental line fitting
  - K-means

Algorithm 15.1: Incremental line fitting by walking along a curve, fitting a line to runs of pixels along the curve, and breaking the curve when the residual is too large

```
Put all points on curve list, in order along the curve
Empty the line point list
Empty the line list
Until there are too few points on the curve
  Transfer first few points on the curve to the line point list
  Fit line to line point list
  While fitted line is good enough
    Transfer the next point on the curve
      to the line point list and refit the line
  end
  Transfer last point(s) back to curve
  Refit line
  Attach line to line list
end
```

Algorithm 15.2: K-means line fitting by allocating points to the closest line and then refitting.

```
Hypothesize k lines (perhaps uniformly at random)
or
Hypothesize an assignment of lines to points
  and then fit lines using this assignment

Until convergence
  Allocate each point to the closest line
  Refit lines
end
```

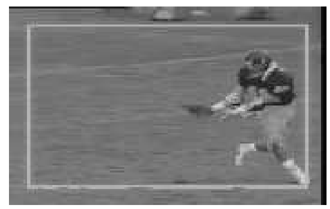
Curve Fitting by Hough Transform

- Let $y=f(x,\mathbf{a})$ be the chosen parameterization of a target curve.
- Discretize the intervals of variation of $a_1,\ldots a_k$ and let $s_1,\ldots s_k$ be the number of the discretized intervals.
- Let $A(s_1,\ldots s_k)$ be an array of integer counters and initialize all its elements to zero.
- For each pixel $E(i,j)$ such that $E(i,j)=1$, increment all counters on the curve defined by $y=f(x,\mathbf{a})$ in A.
- Find all local maxima above certain threshold.

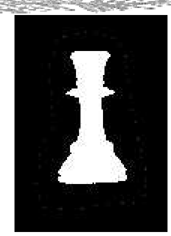
Curve Fitting by Hough Transform

- Suffer with the same problems as line fitting by Hough Transform.
- Computational complexity and storage complexity increase rapidly with number of parameters.
- Not very robust to noise

Deformable Contours



Deformable Contours

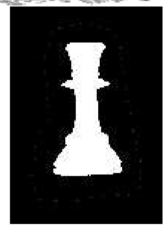


- Minimize the Energy Functional

$$E = \int \left[\alpha(s)E_{cont} + \beta(s)E_{curv} + \gamma(s)E_{image}\right]ds$$

- Where the integral is taken along the contour **c** and each of the energy terms in the functional is a function of **c** or or the derivatives of **c** with respect to *s*. The parameters α, β, and γ control the relative influence of the corresponding energy term, and can vary along **c**.

Deformable Contours



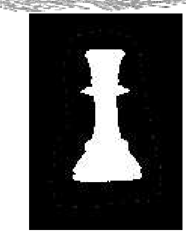
- Continuity

$$E_{cont} = \left\| \frac{d\mathbf{c}}{ds} \right\|^2$$

$$E_{cont} = \left\| p_i - p_{i-1} \right\|^2$$ Discrete Approximation

$$E_{cont} = \left(\bar{d} - \left\| p_i - p_{i-1} \right\| \right)^2$$ A better form

Deformable Contours

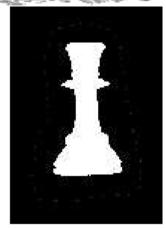


- Curvature (Smoothness)

$$E_{curv} = \left\| \frac{d^2\mathbf{c}}{ds^2} \right\|^2$$

$$E_{cont} = \left\| p_{i-1} - 2p_i + p_{i+1} \right\|^2$$ Discrete Approximation

Deformable Contours



- Image (Edge Attraction)

$$E_{curv} = -\|\Delta I\|$$

Greedy Algorithm (Williams & Shah)

Let I be the intensity image and $p_1,\ldots p_k$ be the initial positions of the snake points.

While a fraction greater than f of the snake points move in an iteration:

1. For each i, find the location of $N(p_i)$ for which the functional is minimum and move the snake point p_i to that location.
2. For each i, estimate the curvature k of the snake and look for local maxima. Set $\beta(j)=0$ for all p_j at which the curvature has a local maximum and is above certain threshold and at which the image gradient is above certain threshold.
3. Update the value of the average distance

Suggested Reading

- Chapter 15, David A. Forsyth and Jean Ponce, "Computer Vision: A Modern Approach“
- Chapter 5, Emanuele Trucco, Alessandro Verri, "Introductory Techniques for 3-D Computer Vision“
- Donna Williams, and Mubarak Shah. “A Fast Algorithm for Active Contours and Curvature Estimation,” Computer Vision, Graphics and Image Processing, Vol 55, No.1, January 1992, pp 14-26.