CAP5415 Computer Vision
Spring 2003

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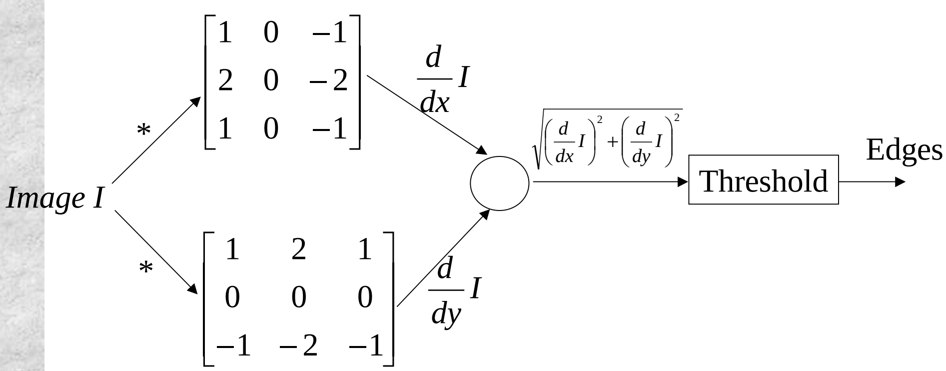
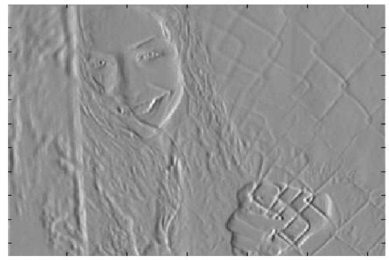
Detecting Edges in Image

- Sobel Edge Detector



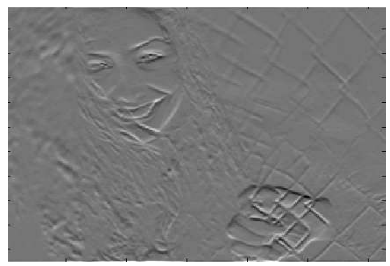
Sobel Edge Detector

$\frac{d}{dx} I$



I

$\frac{d}{dy} I$



Sobel Edge Detector

$$\Delta = \sqrt{\left(\frac{d}{dx} I\right)^2 + \left(\frac{d}{dy} I\right)^2}$$

I

$\Delta \geq Threshold = 100$

Marr and Hildreth Edge Operator

- Smooth by Gaussian

$$S = G_\sigma * I \qquad G_\sigma = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

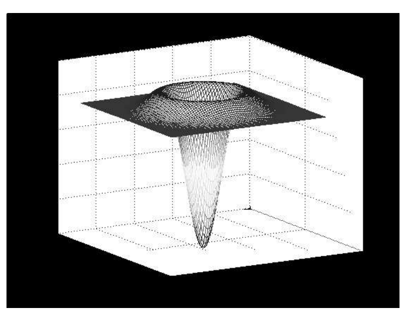
- Use Laplacian to find derivatives

$$\Delta^2 S = \frac{\partial^2}{\partial x^2} S + \frac{\partial^2}{\partial y^2} S$$

Marr and Hildreth Edge Operator

$$\Delta^2 S = \Delta^2 \left(G_\sigma * I \right) = \Delta^2 G_\sigma * I$$

$$\Delta^2 G_\sigma = -\frac{1}{\sqrt{2\pi}\sigma^3} \left(2 - \frac{x^2 + y^2}{\sigma^2} \right) e^{-\frac{x^2+y^2}{2\sigma^2}}$$



Marr and Hildreth Edge Operator

$$\Delta^2 G_\sigma = -\frac{1}{\sqrt{2\pi}\sigma^3}\left(2-\frac{x^2+y^2}{\sigma^2}\right)e^{-\frac{x^2+y^2}{2\sigma^2}}$$

Y

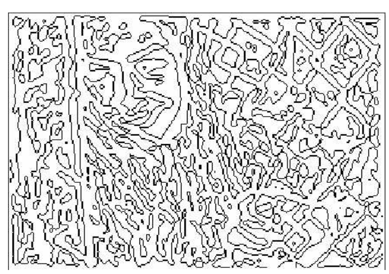
X

0.0008	0.0066	0.0215	0.031	0.0215	0.0066	0.0008
0.0066	0.0438	0.0982	0.108	0.0982	0.0438	0.0066
0.0215	0.0982	0	-0.242	0	0.0982	0.0215
0.031	0.108	-0.242	-0.7979	-0.242	0.108	0.031
0.0215	0.0982	0	-0.242	0	0.0982	0.0215
0.0066	0.0438	0.0982	0.108	0.0982	0.0438	0.0066
0.0008	0.0066	0.0215	0.031	0.0215	0.0066	0.0008

Marr and Hildreth Edge Operator

Image I → $*\Delta^2 G_\sigma$ → $\Delta^2 G_\sigma * I$ → Zero Crossings Detection → Edge Image

$\Delta^2 G_\sigma * I$

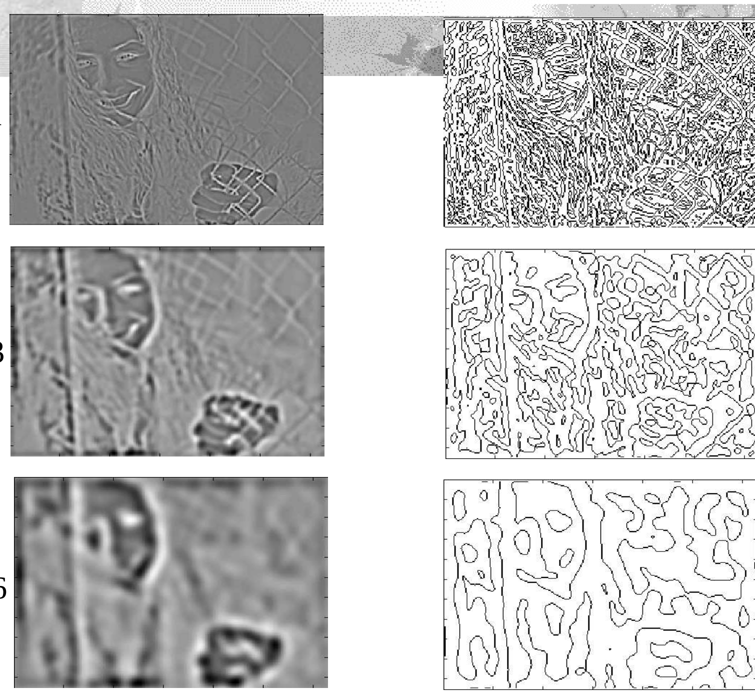


Zero Crossings

$\sigma = 1$

$\sigma = 3$

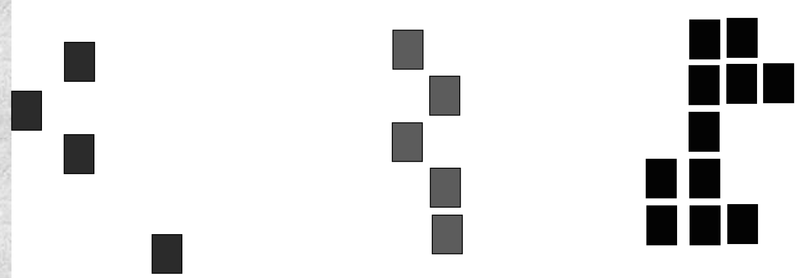
$\sigma = 6$



Quality of an Edge Detector

- Robustness to Noise
- Localization
- Too Many/Too less Responses

True Edge



Poor robustness to noise

Poor localization

Too many responses

Canny Edge Detector

- Criterion 1: Good Detection: The optimal detector must minimize the probability of false positives as well as false negatives.
- Criterion 2: Good Localization: The edges detected must be as close as possible to the true edges.
- Single Response Constraint: The detector must return one point only for each edge point.

Canny Edge Detector

- Difficult to find closed-form solutions.

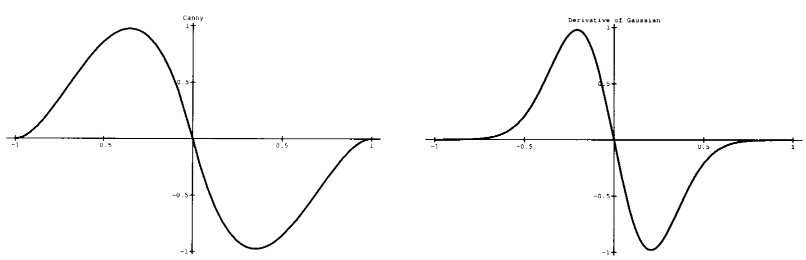


Figure 4.15 A comparison between the Canny operator and the first derivative of a Gaussian.

Canny Edge Detector

- Convolution with derivative of Gaussian
- Non-maximum Suppression
- Hysteresis Thresholding

Canny Edge Detector

- Smooth by Gaussian

$$S = G_\sigma * I \qquad G_\sigma = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

- Compute x and y derivatives

$$\Delta S = \begin{bmatrix} \frac{\partial}{\partial x} S & \frac{\partial}{\partial y} S \end{bmatrix}^T = \begin{bmatrix} S_x & S_y \end{bmatrix}^T$$

- Compute gradient magnitude and orientation

$$|\Delta S| = \sqrt{S_x^2 + S_y^2}$$

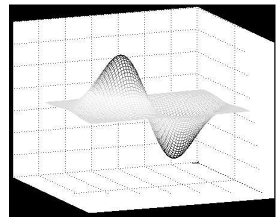
$$\theta = \tan^{-1} \frac{S_y}{S_x}$$

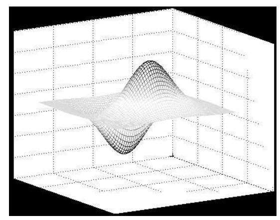
Canny Edge Operator

$$\Delta S = \Delta(G_\sigma * I) = \Delta G_\sigma * I$$

$$\Delta G_\sigma = \left[\frac{\partial G_\sigma}{\partial x} \quad \frac{\partial G_\sigma}{\partial y} \right]^T$$

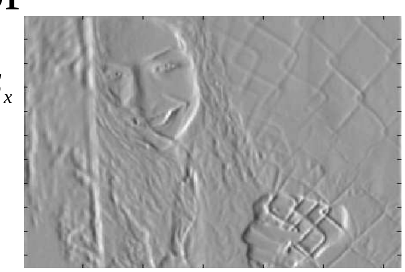
$$\Delta S = \left[\frac{\partial G_\sigma}{\partial x} * I \quad \frac{\partial G_\sigma}{\partial y} * I \right]^T$$





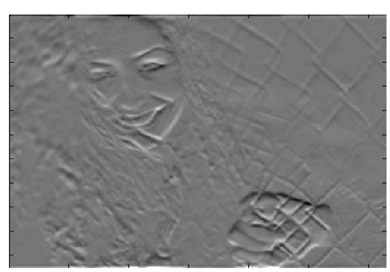
Canny Edge Detector

S_x



I

S_y



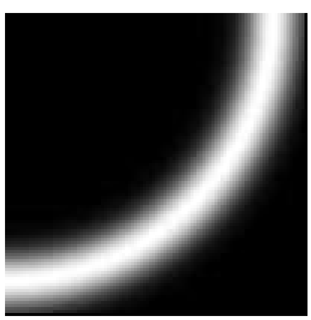
Canny Edge Detector

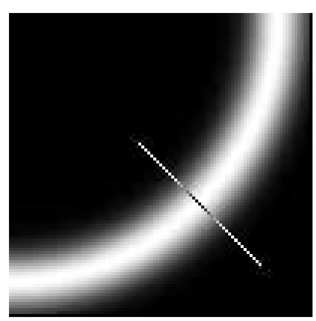
$$|\Delta S| = \sqrt{S_x^2 + S_y^2}$$

I

$$|\Delta S| \geq Threshold = 25$$

Non-Maximum Suppression



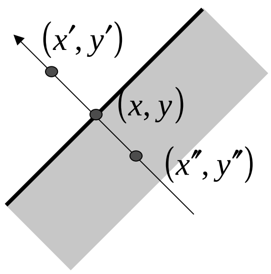


We wish to mark points along the curve where the magnitude is biggest. We can do this by looking for a maximum along a slice normal to the curve (non-maximum suppression). These points should form a curve. There are then two algorithmic issues: at which point is the maximum, and where is the next one?

Non-Maximum Suppression

- Suppress the pixels in 'Gradient Magnitude Image' which are not local maximum

$$M(x,y)=\begin{cases} |\Delta S|(x,y) & \text{if } |\Delta S|(x,y)>|\Delta S|(x',y') \\ & \& \, |\Delta S|(x,y)>|\Delta S|(x'',y'') \\ 0 & \text{otherwise} \end{cases}$$



(x',y') and (x'',y'') are the neighbors of (x,y) in $|\Delta S|$ along the direction normal to an edge

Non-Maximum Suppression

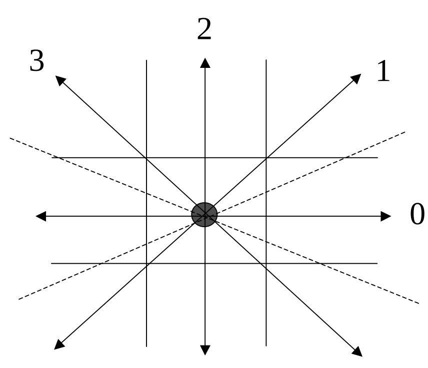
$$\tan\theta = \frac{S_y}{S_x}$$

$0: -0.4142 < \tan\theta \le 0.4142$

$1: 0.4142 < \tan\theta < 2.4142$

$2: |\tan\theta| \ge 2.4142$

$3: -2.4142 < \tan\theta \le -0.4142$

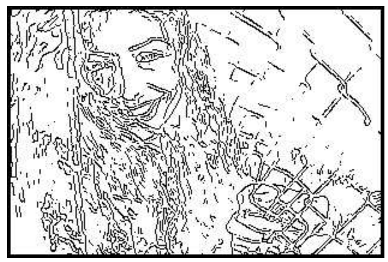


Non-Maximum Suppression

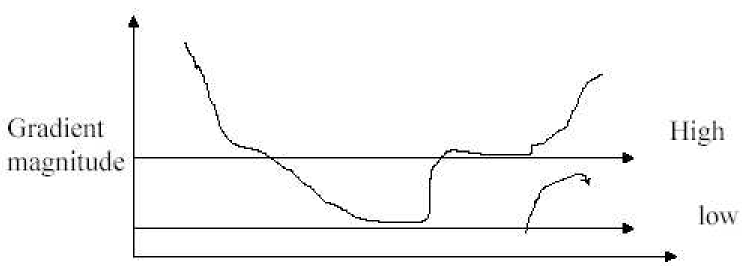
$$|\Delta S| = \sqrt{S_x^2 + S_y^2}$$

M

$M \geq Threshold = 25$



Hysteresis Thresholding

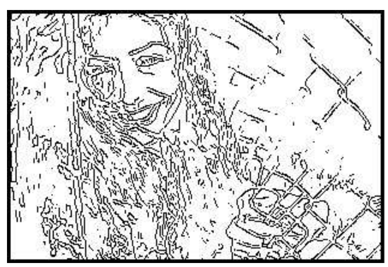


Hysteresis Thresholding

- If the gradient at a pixel is above 'High', declare it an 'edge pixel'
- If the gradient at a pixel is below 'Low', declare it a 'non-edge-pixel'
- If the gradient at a pixel is between 'Low' and 'High' then declare it an 'edge pixel' if and only if it is connected to an 'edge pixel' directly or via pixels between 'Low' and ' High'

Hysteresis Thresholding

M



$M \geq Threshold = 25$

$High = 35$

$Low = 15$

Finding Connected Components

- Scan the binary image left to right top to bottom
- If there is an unlabeled pixel *p* with a value of ‘1’
  - assign a new label to it
  - Recursively check the neighbors of pixel *p* and assign the same label if they are unlabeled with a value of ‘1’.
- Stop when all the pixels with value ‘1’ have been labeled.

Connectedness

Suggested Reading

- Chapter 8, David A. Forsyth and Jean Ponce, "Computer Vision: A Modern Approach“
- Chapter 4, Emanuele Trucco, Alessandro Verri, "Introductory Techniques for 3-D Computer Vision"
- Chapter 2, Mubarak Shah, “Fundamentals of Computer Vision”