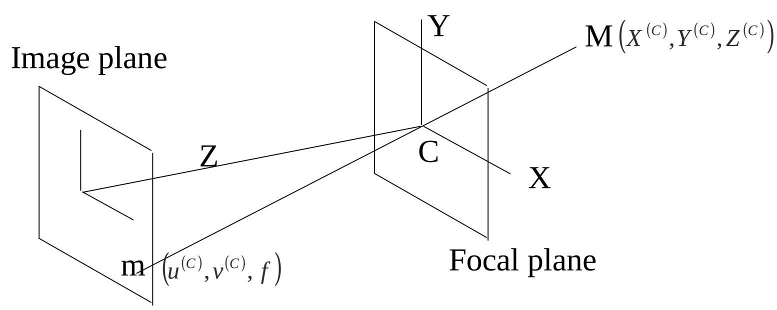
CAP5415 Computer Vision
Spring 2003

Khurram Hassan-Shafique

Camera Parameters



From Perspective Projection

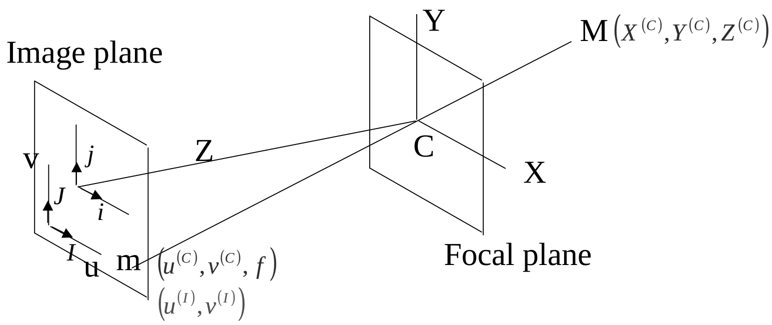
$$u^{(C)} = -f\,\frac{X^{(C)}}{Z^{(C)}} = \frac{U}{S}$$

$$v^{(C)} = -f\,\frac{Y^{(C)}}{Z^{(C)}} = \frac{U}{S}$$

$$\Rightarrow \begin{bmatrix} U \\ V \\ S \end{bmatrix} = \begin{bmatrix} -f & 0 & 0 & 0 \\ 0 & -f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X^{(C)} \\ Y^{(C)} \\ Z^{(C)} \\ 1 \end{bmatrix}$$

Camera Parameters

$$i = k_u I$$
$$j = k_v J$$



$$\begin{bmatrix} u^{(I)} \\ v^{(I)} \\ 1 \end{bmatrix} = \begin{bmatrix} k_u & 0 & 0 \\ 0 & k_v & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u^{(c)} \\ v^{(c)} \\ 1 \end{bmatrix} + \begin{bmatrix} u_0 \\ v_0 \\ 0 \end{bmatrix} = \begin{bmatrix} k_u & 0 & u_0 \\ 0 & k_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u^{(c)} \\ v^{(c)} \\ 1 \end{bmatrix}$$

Camera Parameters

$$\begin{bmatrix} u^{(I)} \\ v^{(I)} \\ 1 \end{bmatrix} = \begin{bmatrix} k_u & 0 & u_0 \\ 0 & k_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u^{(c)} \\ v^{(c)} \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} U^{(new)} \\ V^{(new)} \\ S \end{bmatrix} = \begin{bmatrix} k_u & 0 & u_0 \\ 0 & k_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} U \\ V \\ S \end{bmatrix}$$ —— Equation 3

$$u^{(I)} = \frac{U^{(new)}}{S} \qquad u^{(C)} = \frac{U}{S}$$

$$v^{(I)} = \frac{V^{(new)}}{S} \qquad v^{(C)} = \frac{V}{S}$$

Camera Parameters

Equation 1:
$$\begin{bmatrix} U \\ V \\ S \end{bmatrix} = \begin{bmatrix} -f & 0 & 0 & 0 \\ 0 & -f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X^{(C)} \\ Y^{(C)} \\ Z^{(C)} \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} U^{(new)} \\ V^{(new)} \\ S \end{bmatrix} = \begin{bmatrix} k_u & 0 & u_0 \\ 0 & k_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} U \\ V \\ S \end{bmatrix} = \begin{bmatrix} k_u & 0 & u_0 \\ 0 & k_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -f & 0 & 0 & 0 \\ 0 & -f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X^{(C)} \\ Y^{(C)} \\ Z^{(C)} \\ 1 \end{bmatrix}$$

Camera Parameters

$$\begin{bmatrix} U^{(new)} \\ V^{(new)} \\ S \end{bmatrix} = \begin{bmatrix} -fk_u & 0 & u_0 & 0 \\ 0 & -fk_v & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X^{(C)} \\ Y^{(C)} \\ Z^{(C)} \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} U^{(new)} \\ V^{(new)} \\ S \end{bmatrix} = \begin{bmatrix} -f_u & 0 & u_0 & 0 \\ 0 & -f_v & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X^{(C)} \\ Y^{(C)} \\ Z^{(C)} \\ 1 \end{bmatrix}$$

Equation 4

$f_u = fk_u$
$f_v = fk_v$

Camera Parameters

Intrinsic Parameters (Do not depend on camera position):

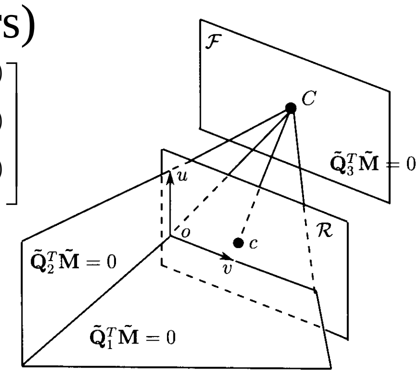
1. $f_u = fk_u$
2. $f_v = fk_v$
3. u_0
4. v_0

Intrinsic Parameters

$$\begin{bmatrix} U^{(new)} \\ V^{(new)} \\ S \end{bmatrix} = \begin{bmatrix} -f_u & 0 & u_0 & 0 \\ 0 & -f_v & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X^{(C)} \\ Y^{(C)} \\ Z^{(C)} \\ 1 \end{bmatrix}$$

$$m^{(I)} = PM^{(C)} = \begin{bmatrix} Q_1^T \\ Q_2^T \\ Q_3^T \end{bmatrix} M^{(C)} = \begin{bmatrix} Q_1^T M^{(C)} \\ Q_2^T M^{(C)} \\ Q_3^T M^{(C)} \end{bmatrix}$$

Calculating Camera Center (from Intrinsic Parameters)



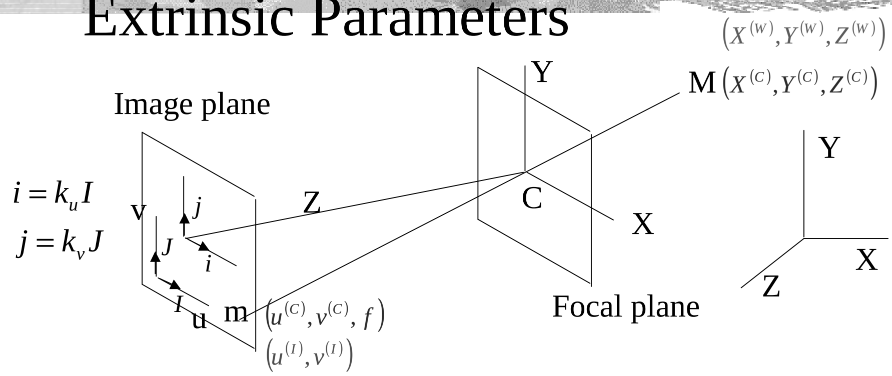
$$m^{(I)} = \begin{bmatrix} Q_1^T \\ Q_2^T \\ Q_3^T \end{bmatrix} M^{(C)} = \begin{bmatrix} Q_1^T M^{(C)} \\ Q_2^T M^{(C)} \\ Q_3^T M^{(C)} \end{bmatrix}$$

$$P\begin{bmatrix} C \\ 1 \end{bmatrix} = \begin{bmatrix} P' & p \end{bmatrix}\begin{bmatrix} C \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow P'C + p = 0$$

$$\Rightarrow C = -(P')^{-1} p$$

Extrinsic Parameters



By Rigid Body Transformation:

$$\begin{bmatrix} X^{(C)} \\ Y^{(C)} \\ Z^{(C)} \\ 1 \end{bmatrix} = \begin{bmatrix} R_{3\times3} & T_{3\times1} \\ 0_{1\times3} & 1 \end{bmatrix} \begin{bmatrix} X^{(W)} \\ Y^{(W)} \\ Z^{(W)} \\ 1 \end{bmatrix} \Rightarrow M^{(C)} = DM^{(W)}$$

Camera Model

$$m^{(I)} = PM^{(C)}, M^{(C)} = DM^{(W)} \Rightarrow m^{(I)} = PDM^{(W)}$$

$$\begin{bmatrix} U^{(new)} \\ V^{(new)} \\ S \end{bmatrix} = \begin{bmatrix} -f_u & 0 & u_0 & 0 \\ 0 & -f_v & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} R_{3\times3} & T_{3\times1} \\ 0_{1\times3} & 1 \end{bmatrix} \begin{bmatrix} X^{(W)} \\ Y^{(W)} \\ Z^{(W)} \\ 1 \end{bmatrix}$$

$$\text{Let } R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \text{ and } T = \begin{bmatrix} T_X \\ T_Y \\ T_Z \end{bmatrix}$$

Camera Model

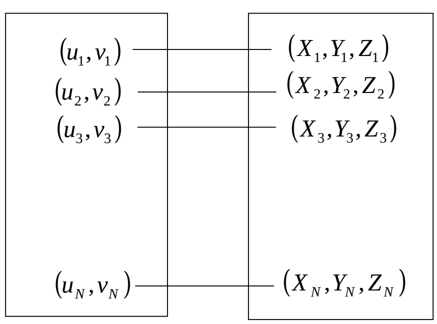
$$u^{(I)} - u_0 = -f_u \frac{r_{11}X^{(W)} + r_{12}Y^{(W)} + r_{13}Z^{(W)} + T_X}{r_{31}X^{(W)} + r_{32}Y^{(W)} + r_{33}Z^{(W)} + T_Z}$$

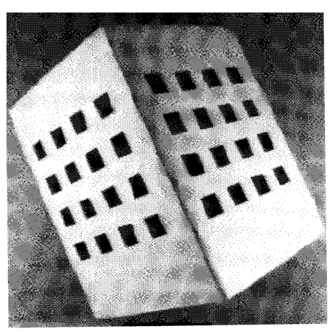
$$v^{(I)} - v_0 = -f_v \frac{r_{21}X^{(W)} + r_{22}Y^{(W)} + r_{23}Z^{(W)} + T_Y}{r_{31}X^{(W)} + r_{32}Y^{(W)} + r_{33}Z^{(W)} + T_Z}$$

Suggested Reading

- Chapter 3, Olivier Faugeras, "Three Dimensional Computer Vision", MIT Press, 1993
- Chapter 2, David A. Forsyth and Jean Ponce, "Computer Vision: A Modern Approach"
- Chapter 1, Mubarak Shah, "Fundamentals of Computer Vision"

Estimating Camera Parameters





Estimating Camera Parameters

For each corresponding pair (u_i, v_i) and (X_i, Y_i, Z_i)

$$u_i^{(I)} - u_0 = -f_u \frac{r_{11}X_i^{(W)} + r_{12}Y_i^{(W)} + r_{13}Z_i^{(W)} + T_X}{r_{31}X_i^{(W)} + r_{32}Y_i^{(W)} + r_{33}Z_i^{(W)} + T_Z}$$

$$v_i^{(I)} - v_0 = -f_v \frac{r_{21}X_i^{(W)} + r_{22}Y_i^{(W)} + r_{23}Z_i^{(W)} + T_Y}{r_{31}X_i^{(W)} + r_{32}Y_i^{(W)} + r_{33}Z_i^{(W)} + T_Z}$$

Let $\alpha = \frac{f_u}{f_v}$ and $u_0 = v_0 = 0$ and dividing the first equation by second

$$u_i^{(I)}\left(r_{21}X_i^{(W)} + r_{22}Y_i^{(W)} + r_{23}Z_i^{(W)} + T_Y\right) = v_i^{(I)}\alpha\left(r_{11}X_i^{(W)} + r_{12}Y_i^{(W)} + r_{13}Z_i^{(W)} + T_X\right)$$

$$u_i^{(I)}X_i^{(W)}r_{21} + u_i^{(I)}Y_i^{(W)}r_{22} + u_i^{(I)}Z_i^{(W)}r_{23} + u_i^{(I)}T_Y - v_i^{(I)}X_i^{(W)}\alpha r_{11} + v_i^{(I)}Y_i^{(W)}\alpha r_{12} + v_i^{(I)}Z_i^{(W)}\alpha r_{13} + v_i^{(I)}\alpha T_X = 0$$

Estimating Camera Parameters

For each corresponding pair (u_i, v_i) and (X_i, Y_i, Z_i)

$$u_i^{(I)}X_i^{(W)}v_1 + u_i^{(I)}Y_i^{(W)}v_2 + u_i^{(I)}Z_i^{(W)}v_3 + u_i^{(I)}v_4 - v_i^{(I)}X_i^{(W)}v_5 + v_i^{(I)}Y_i^{(W)}v_6 + v_i^{(I)}Z_i^{(W)}v_7 + v_i^{(I)}v_8 = 0$$

$v_1 = r_{21}$ $\quad v_5 = \alpha r_{11}$
$v_2 = r_{22}$ $\quad v_6 = \alpha r_{12}$
$v_3 = r_{23}$ $\quad v_7 = \alpha r_{13}$
$v_4 = T_Y$ $\quad v_8 = \alpha T_X$

$$Av = 0$$

$$v = [v_1 \quad v_2 \quad v_3 \quad v_4 \quad v_5 \quad v_6 \quad v_7 \quad v_8]^T$$

Compute *v* by SVD decomposition of *A=UDV*^{T} *(The solution vector is the column of V corresponding to null (or smallest) singular value) in D.*

Estimating Camera Parameters

Computing Scale Factor

Let $\bar{v}$ be the obtained solution vector, then

$$\bar{v} = \gamma\begin{pmatrix} r_{21} & r_{22} & r_{23} & T_Y & \alpha r_{11} & \alpha r_{12} & \alpha r_{13} & \alpha T_X \end{pmatrix}^T$$

Since $r_{21}^2 + r_{22}^2 + r_{23}^2 = 1$, we have

$$\sqrt{v_1^2 + v_2^2 + v_3^2} = \sqrt{\gamma^2\left(r_{21}^2 + r_{22}^2 + r_{23}^2\right)} = |\gamma|$$

Estimating Camera Parameters

Computing α

Since $r_{21}^2 + r_{22}^2 + r_{23}^2 = 1$, we have

$$\sqrt{v_5^2 + v_6^2 + v_7^2} = \sqrt{\gamma^2\alpha^2\left(r_{21}^2 + r_{22}^2 + r_{23}^2\right)} = \alpha|\gamma|$$

Computing Sign of γ

If $x\left(r_{11}X^{(W)} + r_{12}Y^{(W)} + r_{13}Z^{(W)} + T_x\right) > 0$

Reverse the signs

Estimating Camera Parameters

Computing third row of Rotation Matrix

$r_3 = r_1 \times r_2$

Computing f_u, f_v and T_Z

For each corresponding pair (u_i, v_i) and (X_i, Y_i, Z_i)

$$u_i^{(I)}\left(r_{31}X_i^{(W)} + r_{32}Y_i^{(W)} + r_{33}Z_i^{(W)} + T_Z\right) = -f_u\left(r_{11}X_i^{(W)} + r_{12}Y_i^{(W)} + r_{13}Z_i^{(W)} + T_X\right)$$

$$\text{Solve } A\begin{bmatrix} T_Z \\ f_u \end{bmatrix} = b$$

Estimating Camera Parameters

Computing Image Center (u_0, v_0)

Let T be a triangle on the image plane defined by the three Vanishing points of three mutually orthogonal sets of Parallel lines in space. The image center is the orthocenter of T

Suggested Readings:

- Chapter 6, Emanuele Trucco, Alessandro Verri, "Introductory Techniques for 3-D Computer Vision", Prentice Hall, 1998
- Chapter 1, Mubarak Shah, "Fundamentals of Computer Vision"