Xiaochun Cao xccao@cs.ucf.edu

CAP5415 Computer Vision

Assignment # 5

1. Proof:

$$m_n = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$m_n + \frac{1}{n+1}\left(x_{n+1} - m_n\right)$$
$$= \frac{nm_n}{n+1} + \frac{1}{n+1}x_{n+1}$$
$$= \frac{1}{n+1}\sum_{i=1}^{n} x_i + \frac{1}{n+1}x_{n+1}$$
$$= \frac{1}{n+1}\sum_{i=1}^{n+1} x_i$$
$$= m_{n+1}$$

2. Proof:

$$\sigma_n^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - m_n\right)^2$$

$$n\sigma_{n+1}^2 = \sum_{i=1}^{n+1}\left(x_i - m_{n+1}\right)^2$$
$$= \sum_{i=1}^{n+1}\left[\left(x_i - m_n\right) + \left(m_n - m_{n+1}\right)\right]^2$$
$$= \sum_{i=1}^{n+1}\left(x_i - m_n\right)^2 + \sum_{i=1}^{n+1}\left(m_n - m_{n+1}\right)^2 + 2\sum_{i=1}^{n+1}\left(x_i - m_n\right)\left(m_n - m_{n+1}\right)$$
$$= \sum_{i=1}^{n+1}\left(x_i - m_n\right)^2 + (n+1)\left(m_n - m_{n+1}\right)^2 + 2\left(m_n - m_{n+1}\right)\sum_{i=1}^{n+1}\left(x_i - m_n\right)$$
$$= \sum_{i=1}^{n+1}\left(x_i - m_n\right)^2 + (n+1)\left(m_n - m_{n+1}\right)^2 + 2\left(m_n - m_{n+1}\right)(n+1)\left(m_{n+1} - m_n\right)$$
$$= \sum_{i=1}^{n+1}\left(x_i - m_n\right)^2 - (n+1)\left(m_n - m_{n+1}\right)^2 \qquad (1)$$

Since:

$$(n+1)m_{n+1} = nm_n + x_{n+1}$$
$$\Rightarrow x_{n+1} - m_n = (n+1)\left(m_{n+1} - m_n\right) \qquad (2$$

So

$$n\sigma_{n+1}^2 = \sum_{i=1}^{n+1}(x_i - m_n)^2 - (n+1)\frac{1}{(n+1)^2}(x_{n+1} - m_n)^2$$
$$= \sum_{i=1}^{n+1}(x_i - m_n)^2 - \frac{1}{(n+1)}(x_{n+1} - m_n)^2$$
$$= \sum_{i=1}^{n+1}(x_i - m_n)^2 - (x_{n+1} - m_n)^2 + \frac{n}{(n+1)}(x_{n+1} - m_n)^2$$
$$= \sum_{i=1}^{n}(x_i - m_n)^2 + \frac{n}{(n+1)}(x_{n+1} - m_n)^2$$
$$\Rightarrow \sigma_{n+1}^2 = \frac{(n-1)}{n}\sigma_n^2 + \frac{1}{(n+1)}(x_{n+1} - m_n)^2 \tag{3}$$