CAP5415 Computer Vision

Assignment # 2

Let

I be an image,

$$g_\sigma(x)=\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{x^2}{2\sigma^2}\right),\ g_\sigma(x,y)=\frac{1}{2\pi\sigma^2}\exp\left(-\frac{x^2+y^2}{2\sigma^2}\right),$$

$\Delta I=\begin{bmatrix}\frac{\partial}{\partial x}I & \frac{\partial}{\partial y}I\end{bmatrix}^T$ is the Gradient of image *I* and $\Delta^2 I=\frac{\partial^2}{\partial x^2}I+\frac{\partial^2}{\partial y^2}I$ is the Laplacian of image *I*.

$f*g$ denotes the convolution of f with g.

1. Proof the following results
    1. $g_{\sigma_1}(x)*\left(g_{\sigma_2}(x)*I\right)=g_\sigma(x)*I$. Also, find the value of σ in terms of σ_1 and σ_2.
    2. $g_\sigma(x,y)=g_\sigma(x)g_\sigma(y)$.
    3. $g_\sigma(x,y)*I=g_\sigma(x)*\left(g_\sigma(y)*I\right)$.
    4. $\Delta\left(g_\sigma(x,y)*I\right)=\left(\Delta g_\sigma(x,y)\right)*I$.
    5. $|\Delta|$ is isotropic (rotation invariant).
    6. Δ^2 is linear and isotropic (rotation invariant).
    7. $\Delta^2\left(g_\sigma(x,y)*I\right)=\left(\Delta^2 g_\sigma(x,y)\right)*I$.

You may want to use the following results or facts.

1D Convolution: $f*g=\int f(x-x')g(x')dx'$

2D Convolution: $f*g=\int\int f(x-x',y-y')g(x',y')dx'dy'$

Convolution Theorem: $F(f*g)=F(f)F(g)$ and $F(fg)=F(f)*F(g)$, where $F(f)$ is Fourier transform of function f.

Fourier Transform of Gaussian: $F(g_\sigma(x))=\frac{\sqrt{2\pi}}{\sigma}g_{1/\sigma}(u)$, $F(g_\sigma(x,y))=\frac{2\pi}{\sigma^2}g_{1/\sigma}(u,v)$