Lecture-11

Region Properties

Geometrical Properties

Area

$$A = \sum_{x=0}^{m} \sum_{y=0}^{n} B(x, y)$$

Centroid

$$\bar{x} = \frac{\sum_{x=0}^{m} \sum_{y=0}^{n} xB(x, y)}{A}, \quad \bar{y} = \frac{\sum_{x=0}^{m} \sum_{y=0}^{n} yB(x, y)}{A}$$

Moments

General Moments

$$m_{pq} = \iint x^p y^q B(x, y)\,dx\,dy$$

Discrete

$$M_x^1 = \sum_{x=0}^{m} \sum_{y=0}^{n} xB(x, y), \quad M_y^1 = \sum_{x=0}^{m} \sum_{y=0}^{n} yB(x, y)$$

$$M_x^2 = \sum_{x=0}^{m} \sum_{y=0}^{n} x^2 B(x, y), \quad M_y^2 = \sum_{x=0}^{m} \sum_{y=0}^{n} y^2 B(x, y), \quad M_{xy}^2 = \sum_{x=0}^{m} \sum_{y=0}^{n} xyB(x, y)$$

Moments

Central Moments (Translation Invariant)

$$\mu_{pq} = \iint (x - \bar{x})^p (y - \bar{y})^q B(x, y)\; d(x - \bar{x})\,d(y - \bar{y})$$

$$\bar{x} = \frac{m_{10}}{m_{00}}, \quad \bar{y} = \frac{m_{01}}{m_{00}}$$ Centroid

Central Moments

$$\mu_{00} = m_{00} \equiv \mu$$
$$\mu_{01} = 0$$
$$\mu_{10} = 0$$
$$\mu_{20} = m_{20} - \mu\bar{x}^2$$
$$\mu_{11} = m_{11} - \mu\overline{xy}$$
$$\mu_{02} = m_{02} - \mu\bar{y}^2$$
$$\mu_{30} = m_{30} - 3m_{20}\bar{x} + 2\mu\bar{x}^3$$
$$\mu_{21} = m_{21} - m_{20}\bar{y} - 2m_{11}\bar{x} + 2\mu\bar{x}^2 y$$
$$\mu_{12} = m_{12} - m_{02}\bar{x} - 2m_{11}\bar{y} + 2\mu\bar{x}y^2$$
$$\mu_{03} = m_{03} - 3m_{02}\bar{y} + 2\mu\bar{y}^3$$

Moments

Hu Moments: translation, scaling and rotation invariant

$$\phi_1 = \eta_{20} + \eta_{02}$$
$$\phi 2 = (\eta_{20} - \eta_{02})^2 + \eta_{11}{}^2$$
$$\phi_3 = (\eta_{30} - 3\eta_{12})^2 + (3\eta_{12} - \eta_{03})^2$$
$$\phi_4 = (\eta_{30} + \eta_{12})^2 + (\eta_{21} + \eta_{03})^2$$
$$\vdots$$

Half size, mirror
Rotated 2, rotated 45

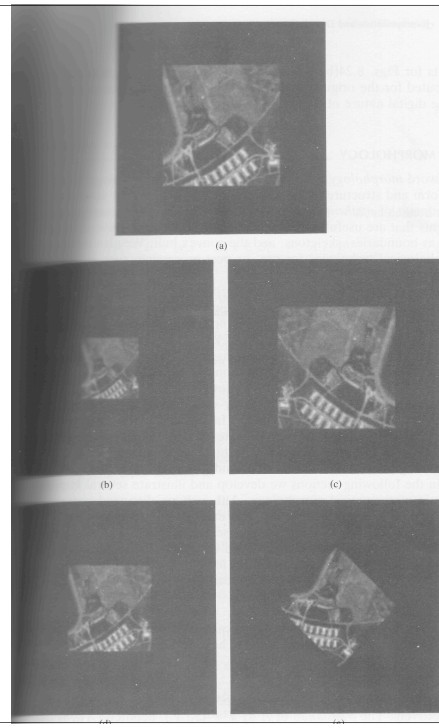


Table 8.2 Moment Invariants for the Images in Figs. 8.24(a)–(e)

Invariant (Log)	*Original*	*Half Size*	*Mirrored*	*Rotated 2°*	*Rotated* [illegible]
ϕ_1	6.249	6.226	6.919	6.253	6.318
ϕ_2	17.180	16.954	19.955	17.270	16.803
ϕ_3	22.655	23.531	26.689	22.836	19.724
ϕ_4	22.919	24.236	26.901	23.130	20.437
ϕ_5	45.749	48.349	53.724	46.136	40.525
ϕ_6	31.830	32.916	37.134	32.068	29.315
ϕ_7	45.589	48.343	53.590	46.017	40.470

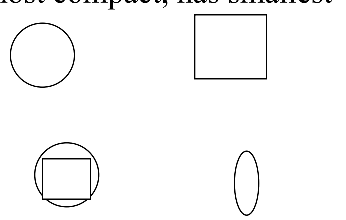
Hu moments

Perimeter & Compactness

Perimeter: The sum of its border points of the region. A pixel which has at least one pixel in its neighborhood from the background is called a border pixel.

Compactness $$C = \frac{P^2}{4\pi A}$$

Circle is the most compact, has smallest value



Area decreases

Orientation of the Region

Least second moment

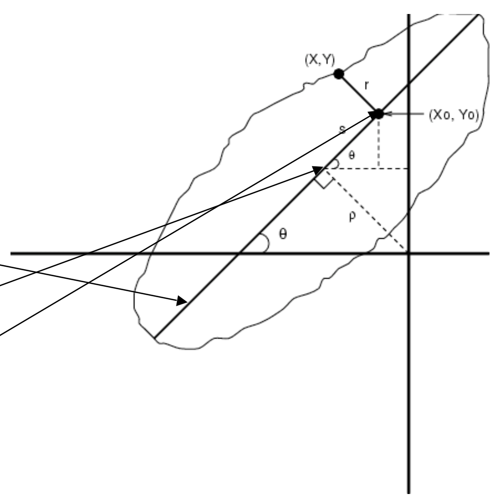
Minimize

$$E = \iint r^2 B(x, y)\,dx\,dy$$

$$x\sin\theta - y\cos\theta + \rho = 0$$

$$(-\rho\sin\theta, \rho\cos\theta)$$

$$x_0 = -\rho\sin\theta + s\cos\theta$$
$$y_0 = \rho\cos\theta + s\sin\theta$$



Orientation of the Region

$$r^2 = (x - x_0)^2 + (y - y_0)^2$$

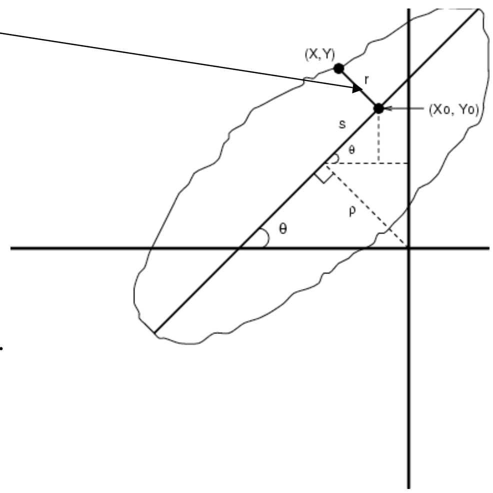
$$x_0 = -\rho \sin\theta + s\cos\theta$$
$$y_0 = \rho\cos\theta + s\sin\theta$$

Substituting (x_0,y_0) in r^2
And differentiating:

$$s = x\cos\theta + y\sin\theta$$

Substitute s in (x_0,y_0), then r:

$$r^2 = (x\sin\theta - y\cos\theta + \rho)^2$$



Orientation of the Region

$$r^2 = (x\sin\theta - y\cos\theta + \rho)^2$$

$$E = \iint r^2 B(x,y)\,dx\,dy$$

$$E = \iint (x\sin\theta - y\cos\theta + \rho)^2 B(x,y)\,dx\,dy$$

Substitute r in E and differentiate
Wrt to ρ and equate it to zero

$$A(\bar{x}\sin\theta - \bar{y}\cos\theta + \rho) = 0$$

is the centroid

$$x' = x - \bar{x},\quad y' = y - \bar{y}$$

Substitute value of

$$a = \iint x'^2 B(x,y)\,dx'\,dy'$$
$$b = \iint x'y' B(x,y)\,dx'\,dy'$$
$$c = \iint y'^2 B(x,y)\,dx'\,dy'$$

$$E = a\sin^2\theta - b\sin\theta\cos\theta + c\cos^2\theta$$

$$E = \frac{1}{2}(a+c) - \frac{1}{2}(a-c)\cos 2\theta - \frac{1}{2}b\sin 2\theta$$

Orientation of the Region

$$E = \frac{1}{2}(a+c) - \frac{1}{2}(a-c)\cos 2\theta - \frac{1}{2}b\sin 2\theta$$

Differentiating this wrt θ

$$\tan 2\theta = \frac{b}{a-c}$$

$$\sin 2\theta = \pm\frac{b}{\sqrt{b^2+(a-c)^2}}$$

$$\cos 2\theta = \pm\frac{a-c}{\sqrt{b^2+(a-c)^2}}$$

$$a = \iint (x')^2 B(x,y)\,dx'\,dy'$$

$$b = \iint x'y' B(x,y)\,dx'\,dy'$$

$$c = \iint (y')^2 B(x,y)\,dx'\,dy'$$

$$x' = x - \bar{x},\quad y' = y - \bar{y}$$

$$a = \sum\sum x^2 B(x,y) - A\bar{x}^2$$

$$b = 2\sum\sum xyB(x,y) - A\overline{xy}$$

$$c = \sum\sum y^2 B(x,y) - A\bar{y}^2$$

Example

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Find: area, centroid, moments, compactness, perimeter, orientation