Lecture-3

Camera Model
Camera Calibration

Camera Model

• Camera is at the origin of the world coordinates first
• Then translated (G),
• then rotated around Z axis in counter clockwise direction,
• then rotated again around X in counter clockwise direction, and
• then translated by C.

$$C_h = PCR^{X}_{\square\square}R^{Z}_{\square\square}GW_h$$

Camera Model

$$C_h = PCR_{\alpha}^{X}R_{\theta}^{Z}GW_h$$

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{f} & 1 \end{bmatrix} \quad R_{\theta}^{Z} = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_{\alpha}^{X} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha & 0 \\ 0 & -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad G = \begin{bmatrix} 1 & 0 & 0 & -X_0 \\ 0 & 1 & 0 & -Y_0 \\ 0 & 0 & 1 & -Z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & -r_1 \\ 0 & 1 & 0 & -r_2 \\ 0 & 0 & 1 & -r_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Camera Model

$$C_h = PCR_{\alpha}^{X}R_{\theta}^{Z}GW_h$$

$$x = \frac{C_{h1}}{C_{h4}}$$

$$y = \frac{C_{h2}}{C_{h4}}$$

$$x = f\frac{(X-X_0)\cos\theta + (Y-Y_0)\sin\theta - r_1}{-(X-X_0)\sin\theta\sin\alpha + (Y-Y_0)\cos\theta\sin\alpha - (Z-Z_0)\cos\alpha + r_3 + f}$$

$$y = f\frac{(X-X_0)\sin\theta\cos\alpha + (Y-Y_0)\cos\theta\cos\alpha + (Z-Z_0)\sin\alpha - r_2}{-(X-X_0)\sin\theta\sin\alpha + (Y-Y_0)\cos\theta\sin\alpha - (Z-Z_0)\cos\alpha + r_3 + f}$$

Camera Model

$$C_h = PCR_{\square\square}^{X} R_{\square\square}^{Z} G W_h$$

$$C_h = AW_h$$

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_3 \\ Ch_4 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$x = \frac{C_{h1}}{C_{h4}}$$

$$y = \frac{C_{h2}}{C_{h4}}$$

$$Ch_1 = a_{11}X + a_{12}Y + a_{13}Z + a_{14} = Ch_4 x$$

$$Ch_2 = a_{21}X + a_{22}Y + a_{23}Z + a_{24} = Ch_4 y$$

$$Ch_4 = a_{41}X + a_{42}Y + a_{43}Z + a_{44}$$

Camera Model

$$Ch_1 = a_{11}X + a_{12}Y + a_{13}Z + a_{14} = Ch_4 x$$

$$Ch_2 = a_{21}X + a_{22}Y + a_{23}Z + a_{24} = Ch_4 y$$

$$Ch_4 = a_{41}X + a_{42}Y + a_{43}Z + a_{44}$$

$$a_{11}X + a_{12}Y + a_{13}Z + a_{14} - a_{41}Xx - a_{42}Yx - a_{43}Zx - a_{44}x = 0$$

$$a_{21}X + a_{22}Y + a_{23}Z + a_{24} - a_{41}Xy - a_{42}Yy - a_{43}Zy - a_{44}y = 0$$

Camera Model

$$a_{11}X + a_{12}Y + a_{13}Z + a_{14} - a_{41}Xx - a_{42}Yx - a_{43}Zx - a_{44}x = 0$$
$$a_{21}X + a_{22}Y + a_{23}Z + a_{24} - a_{41}Xy - a_{42}Yy - a_{43}Zy - a_{44}y = 0$$

One point

$$a_{11}X_1 + a_{12}Y_1 + a_{13}Z_1 + a_{14} - a_{41}X_1x_1 - a_{42}Y_1x_1 - a_{43}Z_1x_1 - a_{44}x_1 = 0$$
$$a_{11}X_2 + a_{12}Y_2 + a_{13}Z_2 + a_{14} - a_{41}X_2x_2 - a_{42}Y_2x_2 - a_{43}Z_2x_2 - a_{44}x_2 = 0$$
$$\vdots$$
$$a_{11}X_n + a_{12}Y_n + a_{13}Z_n + a_{14} - a_{41}X_nx_n - a_{42}Y_nx_n - a_{43}Z_nx_n - a_{44}x_n = 0$$
$$a_{21}X_1 + a_{22}Y_1 + a_{23}Z_1 + a_{24} - a_{41}X_1y_1 - a_{42}Y_1y_1 - a_{43}Z_1y_1 - a_{44}y_1 = 0$$
$$a_{21}X_2 + a_{22}Y_2 + a_{23}Z_2 + a_{24} - a_{41}X_2y_2 - a_{42}Y_2y_2 - a_{43}Z_2y_2 - a_{44}y_2 = 0$$
$$\vdots$$
$$a_{21}X_n + a_{22}Y_n + a_{23}Z_n + a_{24} - a_{41}X_ny_n - a_{42}Y_ny_n - a_{43}Z_ny_n - a_{44}y_n = 0$$

n points
2n equations,
12 unknowns

Camera Model

$$\begin{bmatrix} X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -x_1X_1 & -x_1Y_1 & -x_1Z_1 & -x_1 \\ X_2 & Y_2 & Z_2 & 1 & 0 & 0 & 0 & 0 & -x_2X_2 & -x_2X_2 & -x_2X_2 & -x_2 \\ & & & & & & \vdots & & & & & \\ X_n & Y_n & Z_n & 1 & 0 & 0 & 0 & 0 & -x_nX_n & -x_nY_n & -x_nZ_n & -x_n \\ 0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -y_1X_1 & -y_1X_1 & -y_1Z_1 & -y_1 \\ 0 & 0 & 0 & 0 & X_2 & Y_2 & Z_2 & 1 & -y_2X_2 & -y_2X_2 & -y_2Z_2 & -y_2 \\ & & & & & & \vdots & & & & & \\ 0 & 0 & 0 & 0 & X_n & Y_n & Z_n & 1 & -y_nX_n & -y_nX_n & -y_nX_n & -y_n \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \\ a_{14} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{41} \\ a_{42} \\ a_{43} \\ a_{44} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$CP = 0$$

Camera Model

$$
\begin{bmatrix}
X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -x_1X_1 & -x_1Y_1 & -x_1Z_1 \\
X_2 & Y_2 & Z_2 & 1 & 0 & 0 & 0 & 0 & -x_2X_2 & -x_2X_2 & -x_2X_2 \\
 & & & & & & \vdots & & & & \\
X_n & Y_n & Z_n & 1 & 0 & 0 & 0 & 0 & -x_nX_n & -x_nY_n & -x_nZ_n \\
0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -y_1X_1 & -y_1X_1 & -y_1Z_1 \\
0 & 0 & 0 & 0 & X_2 & Y_2 & Z_2 & 1 & -y_2X_2 & -y_2X_2 & -y_2Z_2 \\
 & & & & & & \vdots & & & & \\
0 & 0 & 0 & 0 & X_n & Y_n & Z_n & 1 & -y_nX_n & -y_nX_n & -y_nX_n
\end{bmatrix}
\begin{bmatrix}
a_{11} \\ a_{12} \\ a_{13} \\ a_{14} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{41} \\ a_{42} \\ a_{43} \\ \
\end{bmatrix}
=
\begin{bmatrix}
x_1 \\ x_2 \\ \vdots \\ x_n \\ y_1 \\ y_2 \\ \vdots \\ y_n
\end{bmatrix}
$$

$$DQ = R$$

$$D^T DQ = D^T R$$

$$Q = (D^T D)^{-1} D^T R$$

Camera Parameters

- Extrinsic parameters
  - Parameters that define the location and orientation of the camera reference frame with respect to a known world reference frame
    - 3-D translation vector
    - A 3 by 3 rotation matrix
- Intrinsic parameters
  - Parameters necessary to link the pixel coordinates of an image point with the corresponding coordinates in the camera reference frame
    - Perspective projection
    - Transformation between camera frame coordinates and pixel coordinates

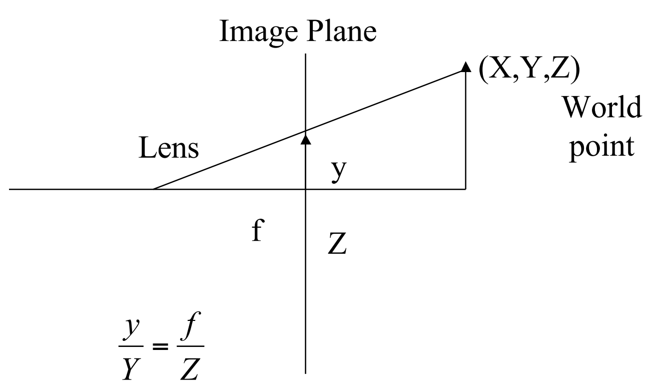
Camera Model Revisited: Rotation & Translation

$$P_c = TRP_w = \begin{bmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & 0 \\ r_{21} & r_{22} & r_{23} & 0 \\ r_{31} & r_{32} & r_{33} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & T_x \\ r_{21} & r_{22} & r_{23} & T_y \\ r_{31} & r_{32} & r_{33} & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$P_c = TRP_w = \begin{bmatrix} r_{11} & r_{12} & r_{13} & T_x \\ r_{21} & r_{22} & r_{23} & T_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$P_c = M_{ext} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Perspective Projection: Revisited



$$\frac{y}{Y} = \frac{f}{Z}$$

$$y = \frac{fY}{Z} \qquad x = \frac{fX}{Z}$$

Camera Model Revisited: Perspective

$$C_h = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$x = \frac{fX}{Z}$$

$$y = \frac{fY}{Z}$$

Origin at the lens
Image plane in front of the lens

Camera Model Revisited: Image and Camera coordinates

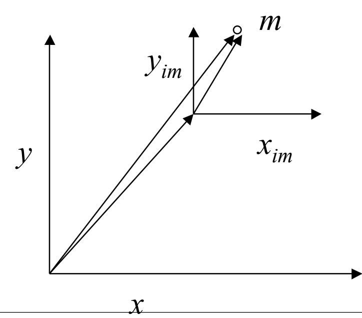
$$x = -(x_{im} - o_x)s_x$$
$$y = -(y_{im} - o_y)s_y$$

$$x_{im} = -\frac{x}{s_x} + o_x$$

$$y_{im} = -\frac{y}{s_y} + o_y$$

$$\begin{bmatrix} x_{im} \\ y_{im} \\ 1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{s_x} & 0 & o_x \\ 0 & -\frac{1}{s_y} & o_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

image coordinates
camera coordinates
image center (in pixels)
effective size of pixels (in millimeters) in the horizontal and vertical directions.



Camera Model Revisited

$$C_h = C \cdot P \cdot T \cdot R \cdot W_h$$

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_4 \end{bmatrix} = \begin{bmatrix} -\frac{1}{s_x} & 0 & o_x \\ 0 & -\frac{1}{s_y} & o_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & T_x \\ r_{21} & r_{22} & r_{23} & T_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_4 \end{bmatrix} = \begin{bmatrix} -\frac{f}{s_x} & 0 & o_x \\ 0 & -\frac{f}{s_x} & o_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & T_x \\ r_{21} & r_{22} & r_{23} & T_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_4 \end{bmatrix} = M_{\text{int}} M_{ext} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_4 \end{bmatrix} = M \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Camera Model Revisited

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_4 \end{bmatrix} = \begin{bmatrix} -\frac{f}{s_x} & 0 & o_x \\ 0 & -\frac{f}{s_y} & o_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & T_x \\ r_{21} & r_{22} & r_{23} & T_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_4 \end{bmatrix} = \begin{bmatrix} -\frac{f}{s_x} r_{11} + r_{31} o_x & -\frac{f}{s_x} r_{12} + r_{32} o_x & -\frac{f}{s_x} r_{13} + r_{33} o_x & -\frac{f}{s_x} T_x + T_z o_x \\ -\frac{f}{s_y} r_{21} + r_{31} o_y & -\frac{f}{s_y} r_{22} + r_{32} o_y & -\frac{f}{s_y} r_{23} + r_{33} o_y & -\frac{f}{s_y} T_y + T_z o_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Camera Model Revisited

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_4 \end{bmatrix} = \begin{bmatrix} \frac{f}{s_x} r_{11} + r_{31} o_x & \frac{f}{s_x} r_{12} + r_{32} o_x & \frac{f}{s_x} r_{13} + r_{33} o_x & \frac{f}{s_x} T_x + T_z o_x \\ \frac{f}{s_y} r_{21} + r_{31} o_y & \frac{f}{s_y} r_{22} + r_{32} o_y & \frac{f}{s_y} r_{23} + r_{33} o_y & \frac{f}{s_y} T_y + T_z o_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

f_x effective focal length expressed in effective horizontal pixel size

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_4 \end{bmatrix} = \begin{bmatrix} f_x r_{11} + r_{31} o_x & f_x r_{12} + r_{32} o_x & f_x r_{13} + r_{33} o_x & f_x T_x + T_z o_x \\ f_y r_{21} + r_{31} o_y & f_y r_{22} + r_{32} o_y & f_y r_{23} + r_{33} o_y & f_y T_y + T_z o_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_4 \end{bmatrix} = M \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Equation 6.18, pp 134

Computing Camera Parameters

- Using known 3-D points and corresponding image points, estimate camera matrix employing pseudo inverse method of section 1.6 (Fundamental of Computer Vision).
- Compute camera parameters by relating camera matrix with estimated camera matrix.
  - Extrinsic
    - Translation
    - Rotation
  - Intrinsic
    - Horizontal and vertical focal lengths
    - Translation o_x and o_y

Comparison

$$\hat{M} = \begin{bmatrix} \hat{m}_{11} & \hat{m}_{12} & \hat{m}_{13} & \hat{m}_{14} \\ \hat{m}_{21} & \hat{m}_{22} & \hat{m}_{23} & \hat{m}_{24} \\ \hat{m}_{31} & \hat{m}_{32} & \hat{m}_{33} & \hat{m}_{34} \end{bmatrix}$$

$$M = \begin{bmatrix} -f_x r_{11} + r_{31} o_x & -f_x r_{12} + r_{32} o_x & -f_x r_{13} + r_{33} o_x & -f_x T_x + T_z o_x \\ -f_y r_{21} + r_{31} o_y & -f_y r_{22} + r_{32} o_y & -f_y r_{23} + r_{33} o_y & -f_y T_y + T_z o_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix}$$

Computing Camera Parameters: Estimating scale

estimated

$$\hat{M} = \gamma M$$

Since M is defined up to a scale factor

$$\begin{bmatrix} \hat{m}_{11} & \hat{m}_{12} & \hat{m}_{13} & \hat{m}_{14} \\ \hat{m}_{21} & \hat{m}_{22} & \hat{m}_{23} & \hat{m}_{24} \\ \hat{m}_{31} & \hat{m}_{32} & \hat{m}_{33} & \hat{m}_{34} \end{bmatrix} = \gamma \begin{bmatrix} -f_x r_{11} + r_{31} o_x & -f_x r_{12} + r_{32} o_x & -f_x r_{13} + r_{33} o_x & -f_x T_x + T_z o_x \\ -f_y r_{21} + r_{31} o_y & -f_y r_{22} + r_{32} o_y & -f_y r_{23} + r_{33} o_y & -f_y T_y + T_z o_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix}$$

$$\sqrt{\hat{m}_{31}^2 + \hat{m}_{32}^2 + \hat{m}_{33}^2} = |\gamma| \sqrt{r_{31}^2 + r_{32}^2 + r_{33}^2} = |\gamma|$$

Divide each entry of $\hat{M}$ by $|\gamma|$.

Computing Camera Parameters: estimating third row of rotation matrix and translation in depth

$$\begin{bmatrix} \hat{m}_{11} & \hat{m}_{12} & \hat{m}_{13} & \hat{m}_{14} \\ \hat{m}_{21} & \hat{m}_{22} & \hat{m}_{23} & \hat{m}_{24} \\ \hat{m}_{31} & \hat{m}_{32} & \hat{m}_{33} & \hat{m}_{34} \end{bmatrix} = \gamma \begin{bmatrix} -f_x r_{11} + r_{31} o_x & -f_x r_{12} + r_{32} o_x & -f_x r_{13} + r_{33} o_x & -f_x T_x + T_z o_x \\ -f_y r_{21} + r_{31} o_y & -f_y r_{22} + r_{32} o_y & -f_y r_{23} + r_{33} o_y & -f_y T_y + T_z o_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix}$$

$$T_z = \gamma \hat{m}_{34}, \quad \gamma = \pm 1$$

$$r_{3i} = \gamma \hat{m}_{3i}, \quad i = 1,2,3$$

Since we can determine $T_z > 0$ *(origin of world reference is in front)*
Or $T_z < 0$ *(origin of world reference is in back)*
we can determine sign.

Computing Camera Parameters

$$\begin{bmatrix} \hat{m}_{11} & \hat{m}_{12} & \hat{m}_{13} & \hat{m}_{14} \\ \hat{m}_{21} & \hat{m}_{22} & \hat{m}_{23} & \hat{m}_{24} \\ \hat{m}_{31} & \hat{m}_{32} & \hat{m}_{33} & \hat{m}_{34} \end{bmatrix} = \gamma \begin{bmatrix} -f_x r_{11} + r_{31} o_x & -f_x r_{12} + r_{32} o_x & -f_x r_{13} + r_{33} o_x & -f_x T_x + T_z o_x \\ -f_y r_{21} + r_{31} o_y & -f_y r_{22} + r_{32} o_y & -f_y r_{23} + r_{33} o_y & -f_y T_y + T_z o_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix}$$

Let

$$q_1 = [\hat{m}_{11} \quad \hat{m}_{12} \quad \hat{m}_{13}]$$
$$q_2 = [\hat{m}_{21} \quad \hat{m}_{22} \quad \hat{m}_{23}]$$
$$q_3 = [\hat{m}_{31} \quad \hat{m}_{32} \quad \hat{m}_{33}]$$

Computing Camera Parameters: origin of image

$$
\begin{aligned}
q_1^T q_3 &= \hat{m}_{11}\hat{m}_{31} + \hat{m}_{12}\hat{m}_{32} + \hat{m}_{13}\hat{m}_{33} \\
\hat{m}_{11}\hat{m}_{31} + \hat{m}_{12}\hat{m}_{32} + \hat{m}_{13}\hat{m}_{33} &= \\
&\left(-f_x r_{11} + r_{31} o_x \quad -f_x r_{12} + r_{32} o_x \quad -f_x r_{13} + r_{33} o_x\right)\left(r_{31} \quad r_{32} \quad r_{33}\right) \\
&= \left(-f_x r_{11} \quad -f_x r_{12} \quad -f_x r_{13}\right)\left(r_{31} \quad r_{32} \quad r_{33}\right) + \\
&\left(r_{31} o_x \quad r_{32} o_x \quad r_{32} o_x\right)\left(r_{31} \quad r_{32} \quad r_{33}\right) \\
&= \left(r_{31} o_x \quad r_{32} o_x \quad r_{32} o_x\right)\left(r_{31} \quad r_{32} \quad r_{33}\right) \\
&= (r_{31}{}^2 o_x + r_{32}{}^2 o_x + r_{33}{}^2 o_x) \\
&= o_x (r_{31}{}^2 + r_{32}{}^2 + r_{33}{}^2) \\
q_1^T q_3 &= o_x
\end{aligned}
$$

Therefore:

$$o_x = q_1^T q_3$$

$$o_y = q_2^T q_3$$

Computing Camera Parameters: vertical and horizontal focal lengths

$$
\begin{aligned}
q_1^T q_1 &= \hat{m}_{11}\hat{m}_{11} + \hat{m}_{12}\hat{m}_{12} + \hat{m}_{13}\hat{m}_{13} \\
\hat{m}_{11}\hat{m}_{11} + \hat{m}_{12}\hat{m}_{12} + \hat{m}_{13}\hat{m}_{13} &= \\
&\left(-f_x r_{11} + r_{31} o_x \quad -f_x r_{12} + r_{32} o_x \quad -f_x r_{13} + r_{33} o_x\right)\left(-f_x r_{11} + r_{31} o_x \quad -f_x r_{12} + r_{32} o_x \quad -f_x r_{13} + r_{33} o_x\right) \\
&= (-f_x r_{11} + r_{31} o_x)^2 + (-f_x r_{12} + r_{32} o_x)^2 + (-f_x r_{13} + r_{33} o_x)^2 \\
&= (f_x{}^2 r_{11}{}^2 + r_{31}{}^2 o_x{}^2) + (f_x{}^2 r_{12}{}^2 + r_{32}{}^2 o_x{}^2) + (f_x{}^2 r_{13}{}^2 + r_{33}{}^2 o_x{}^2) \\
&= f_x{}^2 (r_{11}{}^2 + r_{12}{}^2 + r_{13}{}^2) + o_x{}^2 (r_{31}{}^2 + r_{32}{}^2 + r_{33}{}^2) \\
&= f_x{}^2 + o_x{}^2 \\
q_1^T q_1 &= f_x{}^2 + o_x{}^2 \\
\sqrt{q_1^T q_1 - o_x{}^2} &= f_x
\end{aligned}
$$

Therefore:

$$f_x = \sqrt{q_1^T q_1 - o_x^2}$$

$$f_y = \sqrt{q_2^T q_2 - o_y^2}$$

Computing Camera Parameters: remaining rotation and translation parameters

$$M = \begin{bmatrix} -f_x r_{11} + r_{31} o_x & -f_x r_{12} + r_{32} o_x & -f_x r_{13} + r_{33} o_x & -f_x T_x + T_z o_x \\ -f_y r_{21} + r_{31} o_y & -f_y r_{22} + r_{32} o_y & -f_y r_{23} + r_{33} o_y & -f_y T_y + T_z o_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix}$$

$$r_{1i} = -(0_x \hat{m}_{3i} - \hat{m}_{1i}) / f_x, \quad i = 1,2,3$$
$$r_{2i} = -(0_y \hat{m}_{3i} - \hat{m}_{2i}) / f_y, \quad i = 1,2,3$$
$$T_x = -(0_x T_z - \hat{m}_{14}) / f_x$$
$$T_y = -(0_y T_z - \hat{m}_{24}) / f_y$$

Application

$$M = \begin{bmatrix} -.17237 & -.15879 & .01879 & 274.943 \\ -.131132 & .112747 & .2914 & 258.686 \\ -.000346 & .0003 & .00006 & 1 \end{bmatrix}$$

Camera location: intersection of California and Mason streets, at an elevation of 435 feet above sea level. The camera was oriented at an angle of 8^0 above the horizon. *fs_x=495, fs_y=560.*

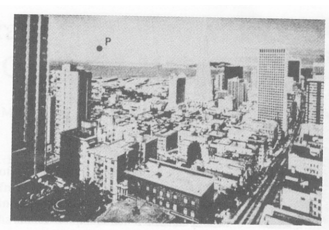


FIGURE 8 PHOTOGRAPH OF SAN FRANCISCO

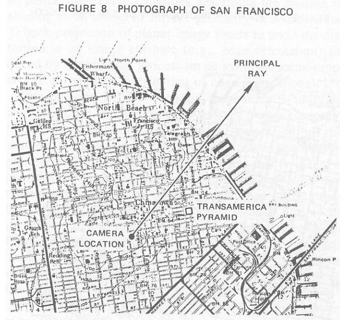


FIGURE 9 MAP OF SAN FRANCISCO

Application

$$M = \begin{bmatrix} -.175451 & -.10520 & .00435 & 297.83 \\ .02698 & -.09635 & .2303 & 249.574 \\ .00015 & -.00016 & .00001 & 1.0 \end{bmatrix}$$

Camera location: at an elevation of 1200 feet above sea level. The camera was oriented at an angle of 4^0 above the horizon. $fs_x=876$, $fs_y=999$.

FIGURE 10 ANOTHER PHOTOGRAPH OF SAN FRANCISCO

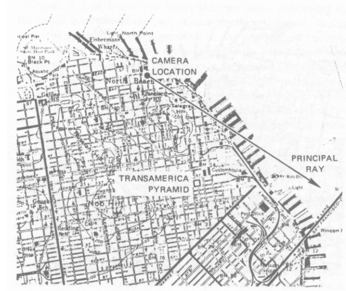


FIGURE 11 MAP OF SAN FRANCISCO