Lecture-6

Mann & Picard

Projective
Projective Flow (weighted)

\[ u_f f_x + v_f f_y + f_t = 0 \]  
Optical Flow const. equation

\[ u_m^T f_x + f_t = 0 \]

Projective transform

\[ x' = \frac{A x + b}{C^T x + 1} \]

\[ x_m = x' - x = \frac{A x + b}{C^T x + 1} \]

Projective Flow (weighted)

\[ \varepsilon_{flow} = \sum (u_m^T f_x + f_t)^2 \]

\[ = \sum ((Ax + b)^T f_x + f_t)^2 \]

\[ = \sum ((Ax + b - (C^T x + 1)x)^T f_x + (C^T x + 1)f_t)^2 \]

minimize
Projective Flow (weighted)

\( (\sum \phi \phi^T) a = \sum (x^T f_x - f_t) \phi \)

\[ a = [a_{11}, a_{12}, b_1, a_{21}, a_{22}, b_2, c_1, c_2]^T \]

\( \phi' = [f_x, f_y, f_z, f_x, f_y, f_z, x f_x - x^2 f_z, x y f_z - x f_y, y f_z - x y f_x - y^2 f_y] \)

Projective Flow (unweighted)
Bilinear

\[ x' = \frac{Ax + b}{C^T x + 1} \]

Taylor Series

\[ u_m + x = a_1 + a_2x + a_5y + a_4xy \]
\[ v_m + y = a_5 + a_6x + a_7y + a_8xy \]

Pseudo-Perspective

\[ x' = \frac{Ax + b}{C^T x + 1} \]

Taylor Series

\[ x + u_m = a_1 + a_2x + a_3y + a_4x^2 + a_5xy \]
\[ y + v_m = a_4 + a_7x + a_8y + a_4xy + a_5y^2 \]
Projective Flow (unweighted)

\[ \varepsilon_{\text{flow}} = \sum (u^T_m f_x + f_t)^2 \]

\[ \downarrow \quad \text{Minimize} \]

Bilinear and Pseudo-Perspective

\[ (\sum \Phi \Phi^T) q = -\sum f_t \Phi \]

\[ \Phi^T = \begin{bmatrix} f_x(xy, x, y, 1) & f_y(xy, x, y, 1) \end{bmatrix} \]

\[ \Phi^T = \begin{bmatrix} f_x(x, y, 1) & f_y(x, y, 1) & c_1 & c_2 \end{bmatrix} \]

bilinear

\[ c_1 = x^2 f_x + x y f_x \]

Pseudo perspective

\[ c_2 = x y f_x + y^2 f_y \]
Algorithm-1

- Estimate “q” (using approximate model, e.g. bilinear model).
- Relate “q” to “p”
  - select four points S1, S2, S3, S4
  - apply approximate model using “q” to compute \((x'_k, y'_k)\)
  - estimate exact “p”:

```
  h
  operation
  \[ \text{est. } q \rightarrow q \rightarrow q_{to} \rightarrow p \]
```
True Projective

\[
x' = \frac{a_1 x + a_2 y + b_1}{c_1 x + c_2 y + 1}
\]

\[
y' = \frac{a_3 x + a_4 y + b_1}{c_1 x + c_2 y + 1}
\]

\[
\begin{bmatrix}
x_k' \\
y_k'
\end{bmatrix} =
\begin{bmatrix}
x_k & y_k & 1 & 0 & 0 & 0 & -x_k x_k' & -y_k x_k' \\
0 & 0 & 0 & x_k & y_k & 1 & -x_k y_k' & -y_k y_k'
\end{bmatrix}
\begin{bmatrix}
a_1 & a_2 & b_1 & a_3 & a_4 & b_2 & c_1 & c_1
\end{bmatrix}^T
\]

\[
a = [a_1, a_2, b_1, a_3, a_4, b_2, c_1, c_1]^T
\]

\[
\begin{bmatrix}
x_k' \\
y_k'
\end{bmatrix} =
\begin{bmatrix}
x_k & y_k & 1 & 0 & 0 & 0 & -x_k x_k' & -y_k x_k' \\
0 & 0 & 0 & x_k & y_k & 1 & -x_k y_k' & -y_k y_k'
\end{bmatrix}
\begin{bmatrix}
a_1 & a_2 & b_1 & a_3 & a_4 & b_2 & c_1 & c_1
\end{bmatrix}
\]

\[
P = Aa
\]

Perform least squares fit to compute a.
Final Algorithm

• A Gaussian pyramid of three or four levels is constructed for each frame in the sequence.
• The parameters “p” are estimated at the top level of the pyramid, between the two lowest resolution images, “g” and “h”, using algorithm-1.

Final Algorithm

• The estimated “p” is applied to the next higher resolution image in the pyramid, to make images at that level nearly congruent.
• The process continues down the pyramid until the highest resolution image in the pyramid is reached.
Video Mosaics

- Mosaic aligns different pieces of a scene into a larger piece, and seamlessly blend them.
  - High resolution image from low resolution images
  - Increased filed of view

Steps in Generating A Mosaic

- Take pictures
- Pick reference image
- Determine transformation between frames
- Warp all images to the same reference view
Applications of Mosaics

• Virtual Environments
• Computer Games
• Movie Special Effects
• Video Compression

Steve Mann
Sequence of Images

Projective Mosaic
Affine Mosaic

Building
Scientific American Frontiers

Head-mounted Camera at Restaurant
MIT Media Lab

Webpages

• http://n1nlf1.eecg.toronto.edu/tip.ps.gz
  Video Orbits of the projective group, S. Mann and R. Picard.

• http://wearcam.org/pencigraphy
  (C code for generating mosaics)
Webpages

• http://ww-bcs.mit.edu/people/adelson/papers.html

Webpages

• http://www.cs.cmu.edu/afs/cs/project/cil/ftp/html/v-source.html (c code for several optical flow algorithms)

• ftp://csd.uwo.ca/pub/vision
  Performance of optical flow techniques (paper)
  Barron, Fleet and Beauchermin
Webpages


• http://www.wisdom.weizmann.ac.il/~irani/abstracts/videoIndexing.html