Symbol Recognition in Sketch-Based Interfaces

Lecture #9: Symbol Recognition
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Recall Pen-Based Interface Dataflow

Raw Stroke Data → Preprocessing → Segmentation → Feature Extraction And Analysis

Sketch Understanding → Ink Parsing → Classification → Make Inferences
Symbol Recognition

- Want to recognize handwritten symbols
  - characters
  - shapes
  - gestures
- Use machine learning approach
- Which algorithm?
  - depends on number of symbols in alphabet
  - complexity (i.e., similarity of symbols)
  - distribution assumptions

Recognition Algorithms

- Many different approaches
- Machine learning techniques (classification)
  - linear classifiers
  - k-means classifiers
  - neural networks
  - Hidden Markov Models
  - template matching
  - support vector machines
  - AdaBoost
- Curve matching
  - elastic matching
- Primitive decomposition
Rubine’s Gesture Recognition Algorithm (Rubine 1991)

- Simple linear classifier
- Utilizes rejection metrics
- Assumes normality for features
- Simple to implement
- Does not need a lot of training samples

Recall Rubine’s Feature Set

- Cosine and sine of initial angle
- Length and angle of bounding box diagonal
- Distance between first and last point
- Cosine and sine of angle between first and last point
- Total gesture length
- Total angle traversed
- Sum of absolute value of the angle at each point
- Sum of squared values of the angle at each point
- Maximum speed
- Stroke duration
Rubine Classifier

\[ v_{\hat{c}} = w_{\hat{c}0} + \sum_{i=1}^{F} w_{\hat{c}i} f_i \quad 0 \leq c < C \]

where \( F \) is the number of features,
\( w_{\hat{c}} \) is the weights, and the classification of symbol \( g \) is the \( c \) that maximizes \( v_{\hat{c}} \).

- Evaluate each gesture \( 0 \leq c < C \).
- \( v_{\hat{c}} = \text{value} = \text{goodness of fit for that gesture } c. \)

Rubine Classifier Training

- Collect \( E \) samples for each symbol class
- Calculate feature vector for each sample for each class
  - \( f_{\hat{c}ei} \) = the feature value of the \( j \)th feature for the \( e \)th sample of
    the \( c \)th symbol
- For each symbol calculate the mean value for each feature

\[ \overline{f_{\hat{c}i}} = \frac{1}{E_{\hat{c}}} \sum_{e=0}^{E_{\hat{c}}-1} f_{\hat{c}ei} \quad \text{where } 0 \leq e < E_{\hat{c}} \]

and \( E_{\hat{c}} \) is the number of training samples per class
Rubine Classifier – Computing Weights

- We first need the covariance matrix of each class $c$

$$
\sum_{\hat{c}ij} = \frac{1}{E_{\hat{c}} - 1} \sum_{c=0}^{C-1} (f_{\hat{c}ei} - \bar{f}_{\hat{c}i})(f_{\hat{c}ej} - \bar{f}_{\hat{c}j})
$$

Rubine Classifier – Computing Weights (2)

- Using the covariance matrices from each class, find the common covariance matrix
  - numerator = non-normalize total covariance
  - denominator = normalization factor = total number of examples – total number of shapes

$$
\sum_{ij} = \frac{\sum_{\hat{c}} \sum_{\hat{c}ij}}{C - 1} - C + \sum_{\hat{c}} E_{\hat{c}}
$$
Rubine Classifier – Computing Weights (3)

- Using the common covariance matrix and the mean feature vectors from each class, we can compute the weights

\[ w_{\hat{c}i} = \sum_{i=1}^{F} (\Sigma^{-1})_{ij} \bar{f}_{\hat{c}i}, \quad 1 \leq j \leq F \]

\[ w_{\hat{c}0} = -\frac{1}{2} \sum_{i=1}^{F} w_{\hat{c}i} \bar{f}_{\hat{c}i} \]

Rubine Classifier – Rejection Measures

- Linear classifier always will classify a symbol as one of the C classes
  - want to try to reject outliers and ambiguous symbols
  - two approaches
    - probabilistic
    - distance measure
Rubine Classifier – Probabilistic Rejection Measure

- Given a symbol \( g \) with feature vector \( f \) classified as class \( i \) (\( v_i > v_j, \forall j \neq i \))

\[
\tilde{P}(i \mid g) = \frac{1}{\sum_{j=0}^{C-1} e^{(v_j - v_i)}}
\]

Reject symbols with \( \tilde{P}(i \mid g) < 0.95 \)

Rubine Classifier – Rejection based on Distance

- Mahalanobis distance – the number of standard deviations a symbol \( g \) is away from the mean of its chosen class \( i \)

\[
\delta^2 = \sum_{j=1}^{F} \sum_{k=1}^{F} (\Sigma^{-1})_{jk} (f_j - \bar{f}_j)(f_k - \bar{f}_k)
\]

Rejecting symbols for which \( \delta^2 > \frac{1}{2} F^2 \)

- May need to be careful not to reject too many good symbols (a simple alternate list to correct mistakes will be helpful)
AdaBoost (Schapire 1997)

- Not really a classification algorithm – more like a framework
- Can use many different classification algorithms within AdaBoost framework
- Works with series of weak (base) classifiers
  - Want to increase the importance of incorrectly classified examples
    - series of weak hypotheses and weights form a strong hypothesis
    - need to ensure weak learners output either 1 or -1
- Many different variants (M1, M2, etc…)

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AdaBoost Algorithm

Given \((x_1, y_1), \ldots, (x_m, y_m)\) where \(x_i \in X, y_i \in Y = \{-1, +1\}\)

Initialize \(D_1(i) = 1/m\)

For \(t = 1\) to \(T\)

- Train weak learner using distribution on \(D_t\)
- Get weak hypothesis \(h_t : X \rightarrow \{-1, +1\}\) with error
  \[ e_t = \text{Pr}_{\text{y} \sim D_t} [h_t(x) \neq y] = \sum_i D_t(i) \]
- Compute \(\alpha_t = \frac{1}{2 \ln \left( \frac{1 - e_t}{e_t} \right)}\)
- Update \(D_{t+1}(i) = \frac{D_t(i)e^{-\alpha_t \cdot w_h(x_i)}}{Z_t}\)

Final hypothesis is \(H(x) = \text{sign}\left( \sum_{t=1}^T \alpha_t h_t(x) \right)\)
More Information on Machine Learning

Readings