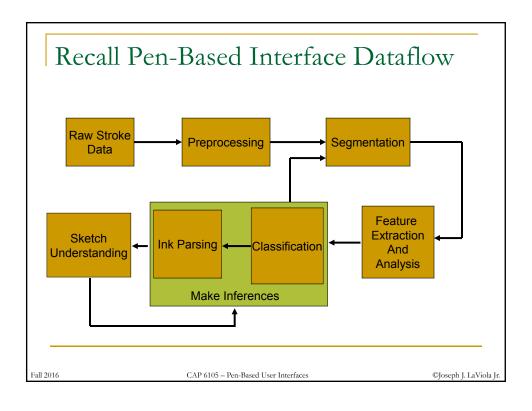
# Features Extraction for Sketch-Based Recognition

Lecture #8: Feature Extraction Joseph J. LaViola Jr. Fall 2016

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## Feature Extraction and Analysis

- What came first, the feature or the machine learning algorithm?
- Want to distinguish sketch components from one another
- Good features are critical
- Extract important information
  - geometrical, statistical, contextual
- Examples include
  - arc length, histograms, cusps, aspect ratio
  - self-intersections, stroke area, etc...

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#### Finding Features

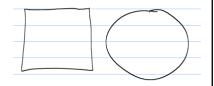
- Challenging problem
  - need fast algorithms for gathering information
  - features must be good discriminators
- Often trial and error
- Can be domain specific

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# Geometric Features (1)

- Number of strokes
  - if you know how many strokes a symbol has, you can break up your recognizer into pieces (i.e., recognizer for 1 stroke symbols, recognizer for 2 stroke symbols ...)
- Cusps
  - smooth vs. jagged strokes
  - distance between cusps
    - useful for when cusps are close together/far apart



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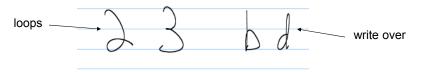
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#### Geometric Features (2)

- Aspect ratio (width / height)
  - □ tall vs. flat



- Self Intersections
  - □ loops vs. no loops
  - strokes with write over
  - distance between self intersections also useful
  - use line segment intersection algorithm



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## Geometric Features (3)

- First and last distance
  - Strokes where first and last points are close together vs. far apart
  - simple computation  $-\|p_n p_1\|$



- Arc length
  - many different symbols have varying arc lengths
  - simple computation as well –

$$l = \sum_{i=2}^{n} ||p_i - p_{i-1}||$$

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#### Geometric Features (4)

- Stroke area
  - area defined by the vectors created with the initial stroke point and consecutive stroke points.
  - good discriminator for straight vs. curved lines

Given 
$$\vec{u}_i = p_{i+1} - p_1$$
 and  $\vec{v}_i = p_{i+2} - p_1$ 

$$s_{area} = \sum_{i=1}^{n-2} \frac{1}{2} (\vec{\mathbf{u}}_{i} \times \vec{\mathbf{v}}_{i}) \cdot \operatorname{sgn}(\vec{\mathbf{u}}_{i} \times \vec{\mathbf{v}}_{i})$$

where  $\vec{\mathbf{u}}_{i} \times \vec{v}_{i}$  is a scalar

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#### Geometric Features (5)

- Fit line feature
  - sophisticated approach to finding how close a stroke is to a straight line
  - finds a least-squares approximation to a line using principal components and then uses this approximation to find the distance of the projection of the stroke points onto the approximated line
  - outputs a value in [0,1]
- What is another name for this approach?

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#### Fit Line Feature Implementation

```
_{
m else}
Input: A set of stroke points P.
                                                                                             if xy_2 = 0
                                                                              (19)
Output: A distance measure
                                                                              (20)
                                                                                                  a \leftarrow b \leftarrow c \leftarrow 0
FITLINE(P)
(1) x_1 \leftarrow \sum_{i=1}^n X(P_i)

(2) y_1 \leftarrow \sum_{i=1}^n Y(P_i)

(3) x_2 \leftarrow \sum_{i=1}^n X(P_i)^2

(4) y_2 \leftarrow \sum_{i=1}^n Y(P_i)^2

(5) xy_1 \leftarrow \sum_{i=1}^n X(P_i)Y(P_i)
                                                                              (21)
                                                                                                  error \leftarrow +\infty
                                                                             (22)
                                                                                             else
                                                                             (23)
                                                                              (24)
                                                                                       mag \leftarrow \sqrt{a^2 + b^2}
                                                                             (25)
                                                                                                (-ax_1-by_1)/n
                                                                             (26)
           x_3 \leftarrow x_2 - x_1^2/n
                                                                             (27)
           y_3 \leftarrow y_2 - y_1^2/n
                                                                             (28)
(8)
           xy_2 \leftarrow xy_1 - (x_1y_1)/n
                                                                             (29)
                                                                                        min_1 \leftarrow +\infty
           rad \leftarrow \sqrt{(x_3 - y_3)^2 + 4xy_2^2}
(9)
                                                                             (30)
                                                                                       max_1 \leftarrow -\infty
(10)
          error \leftarrow (x_3 + y_3 - rad)/2
                                                                                       for i=1 to n
                                                                              (31)
          rms \leftarrow \sqrt{error/n}
                                                                                             err \leftarrow aX(P_i) + bY(P_i) + c
                                                                             (32)
(12)
          if x_3 > y_3
                                                                             (33)
                                                                                             pX \leftarrow X(P_i) - a \cdot err
(13)
                a \leftarrow -2xy_2
                                                                                             pY \leftarrow Y(P_i) - b \cdot err
                                                                             (34)
(14)
                b \leftarrow x_3 - y_3 + rad
                                                                             (35)
                                                                                            ploc \leftarrow -b \cdot pX + b \cdot pY
(15)
           else if x_3 < y_3
                                                                              (36)
                                                                                             min_1 \leftarrow \min(min_1, ploc)
                                                                                       max_1 \leftarrow \max(max_1, ploc) 
 \mathbf{return} \ \frac{100 \cdot rms}{max - min} 
(16)
                a \leftarrow y_3 - x_3 + rad
                                                                              (37)
(17)
                b \leftarrow -2xy_2
                                                                              (38)
```

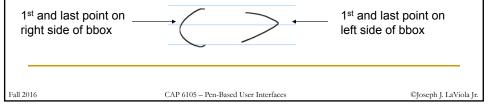
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#### Statistical Features (1)

#### Side ratios

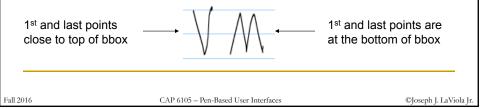
- first and last point of strokes have variable locations with respect to the bounding box
- Approach
  - take the x coordinates of the first and last point of a stroke
  - subtract them from the left side of the symbol's bounding box (i.e., the bounding box's leftmost x value)
  - divide by the bounding box width.



#### Statistical Features (2)

#### Top and Bottom ratios

- similar to side ratios except we are dealing with y coordinate
- approach
  - take y coordinate of the first and last point of a stroke
  - subtract from the top of the symbol's bounding box (i.e., the bounding box's topmost y value)
  - these values are divided by the bounding box height.



## Statistical Features (3)

- Point Histogram
  - distribution of point locations in stroke bounding box
  - discrimination where point concentrations are high
  - approach
    - break up box into n x m grid
    - Count number of points in each sub box
    - divide by total number of points



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#### Statistical Features (4)

- Angle Histogram
  - similar to point histogram except dealing with angles
  - Approach

Given 
$$\vec{\mathbf{v}}_{j} = p_i - p_{i-1}$$
 for  $2 \le i \le n$  and  $\vec{\mathbf{x}} = (1,0)$ 

$$\alpha_{j} = \arccos\left(\vec{x} \cdot \frac{\vec{v}_{j}}{\left\|\vec{v}_{j}\right\|}\right)$$

put angles into bins of n degrees

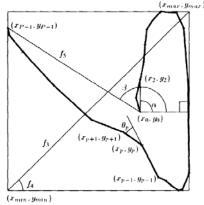


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# The Rubine Feature Set (Rubine 1991)

- Part of Rubine's gesture recognition system
  - we will see this next class
- Stroke
  - P = total number of points
  - p = middle point
  - $\Box$  first point  $(x_0, y_0, t_0)$
  - $\Box$  last point  $(x_{P-1}, y_{P-1}, t_{P-1})$



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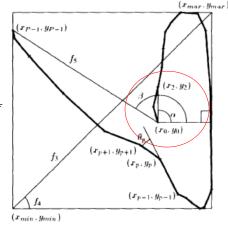
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## Feature f<sub>1</sub>

Cosine of starting angle

$$f_1 = \cos(\alpha) = \frac{(x_2 - x_0)}{\sqrt{(x_2 - x_0)^2 + (y_2 - y_0)^2}}$$



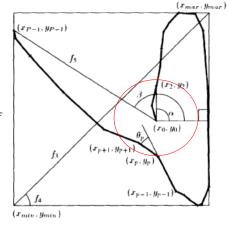
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# Feature f<sub>2</sub>

Sine of starting angle

$$f_2 = \sin(\alpha) = \frac{(y_2 - y_0)}{\sqrt{(x_2 - x_0)^2 + (y_2 - y_0)^2}}$$



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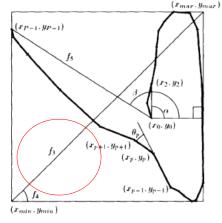
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# Feature f<sub>3</sub>

$$f_3 = \sqrt{(x_{\text{max}} - x_{\text{min}})^2 + (y_{\text{max}} - y_{\text{min}})^2}$$

 Length of diagonal of bounding box (gives an idea of the size of the bounding box)



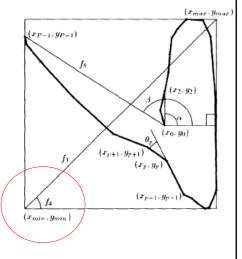
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# Feature f<sub>4</sub>

- Angle of diagonal
- gives an idea of the shape of the bounding box (long, tall, square)

$$f_4 = \arctan\left(\frac{y_{\text{max}} - y_{\text{min}}}{x_{\text{max}} - x_{\text{min}}}\right)$$



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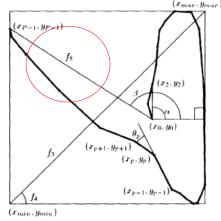
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## Feature f<sub>5</sub>

$$f_5 = \sqrt{(x_{P-1} - x_0)^2 + (y_{P-1} - y_0)^2}$$

 Distance from start to end of stroke



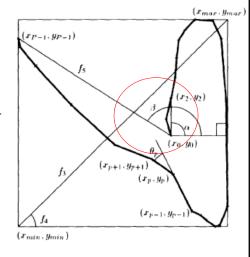
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# Feature f<sub>6</sub>

Cosine of ending angle

$$f_6 = \cos(\beta) = \frac{(x_{P-1} - x_0)}{f_5}$$



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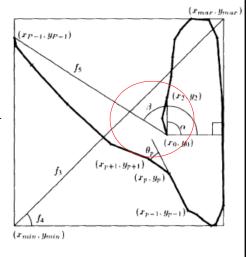
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# Feature f<sub>7</sub>

Sine of ending angle

$$f_7 = \sin(\beta) = \frac{(x_{P-1} - x_0)}{f_5}$$



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## More Definitions (before we continue)

Let 
$$\Delta x_p = x_{p+1} - x_p$$
 and  $\Delta y_p = y_{p+1} - y_p$ 

$$\mathrm{Let}\,\theta_p = \mathrm{arctan}\,\frac{\Delta x_p \Delta y_{p-1} - \Delta x_{p-1} \Delta y_p}{\Delta x_p \Delta x_{p-1} + \Delta y_p \Delta y_{p-1}} \quad \mathrm{^{Directional}}_{\mathrm{angle}}$$

$$\label{eq:left_def} \operatorname{Let} \Delta t_p = t_{p+1} - t_p \quad \text{ Time delta}$$

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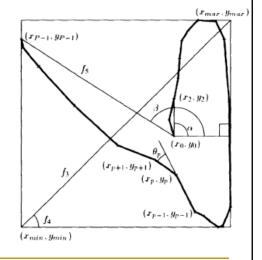
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## Feature f<sub>8</sub>

Total stroke length

$$f_8 = \sum_{p=0}^{P-2} \sqrt{\Delta x_p^2 + \Delta y_p^2}$$



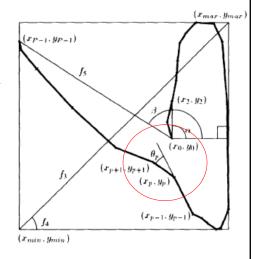
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# Feature f<sub>9</sub>

- Total rotation (from start to end point)
- (not the same as β-α think of spirals)

$$f_9 = \sum_{p=1}^{P-2} \theta_p$$



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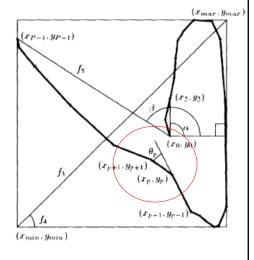
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## Feature f<sub>10</sub>

- Absolute rotation
- How much does it move around

$$f_{10} = \sum_{p=1}^{P-2} \left| \theta_p \right|$$



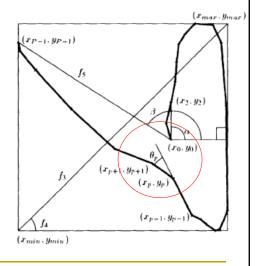
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# Feature f<sub>11</sub>

- Rotation squared
- How smooth are the turns?
- Measure of sharpness

$$f_{11} = \sum_{p=1}^{P-2} \theta_p^2$$



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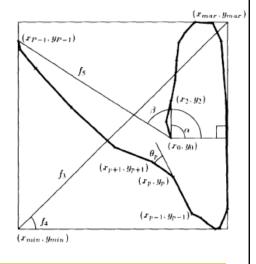
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## Feature f<sub>12</sub>

The maximum speed reached (squared)

$$f_{12} = \max_{p=0}^{P-2} \frac{\Delta x_p^2 + \Delta y_p^2}{\Delta t_p^2}$$



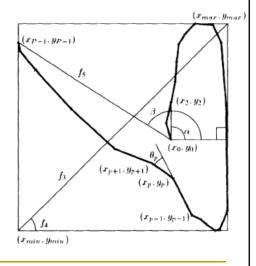
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# Feature f<sub>13</sub>

Total time of stroke

$$f_{13} = t_{P-1} - t_0$$



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## Next Class

Papers Discussion then Symbol Recognition

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