Features Extraction for Sketch-Based Recognition

Lecture #8: Feature Extraction
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Recall Pen-Based Interface Dataflow

- Raw Stroke Data
- Preprocessing
- Segmentation
- Ink Parsing
- Classification
- Make Inferences
- Feature Extraction And Analysis

Sketch Understanding
Feature Extraction and Analysis

- What came first, the feature or the machine learning algorithm?
- Want to distinguish sketch components from one another
- Good features are critical
- Extract important information
  - geometrical, statistical, contextual
- Examples include
  - arc length, histograms, cusps, aspect ratio
  - self-intersections, stroke area, etc...

Finding Features

- Challenging problem
  - need fast algorithms for gathering information
  - features must be good discriminators
- Often trial and error
- Can be domain specific
Geometric Features (1)

- **Number of strokes**
  - if you know how many strokes a symbol has, you can break up your recognizer into pieces (i.e., recognizer for 1 stroke symbols, recognizer for 2 stroke symbols …)

- **Cusps**
  - smooth vs. jagged strokes
  - distance between cusps
    - useful for when cusps are close together/far apart

Geometric Features (2)

- **Aspect ratio (width / height)**
  - tall vs. flat

- **Self Intersections**
  - loops vs. no loops
  - strokes with write over
  - distance between self intersections also useful
  - use line segment intersection algorithm

 loops \[ \rightarrow \]
 write over \[ \rightarrow \]
Geometric Features (3)

- First and last distance
  - Strokes where first and last points are close together vs. far apart
  - Simple computation – \( \| p_n - p_1 \| \)

- Arc length
  - Many different symbols have varying arc lengths
  - Simple computation as well –

\[
l = \sum_{i=2}^{n} \| p_i - p_{i-1} \|
\]

Geometric Features (4)

- Stroke area
  - Area defined by the vectors created with the initial stroke point and consecutive stroke points.
  - Good discriminator for straight vs. curved lines

Given \( \vec{u}_i = p_{i+1} - p_i \) and \( \vec{v}_i = p_{i+2} - p_i \)

\[
s_{area} = \sum_{i=1}^{n-2} \frac{1}{2} (\vec{u}_i \times \vec{v}_i) \cdot \text{sgn}(\vec{u}_i \times \vec{v}_i)
\]

where \( \vec{u}_i \times \vec{v}_i \) is a scalar
Geometric Features (5)

- **Fit line feature**
  - A sophisticated approach to finding how close a stroke is to a straight line.
  - Finds a least-squares approximation to a line using principal components and then uses this approximation to find the distance of the projection of the stroke points onto the approximated line.
  - Outputs a value in [0, 1]

- What is another name for this approach?

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**Fit Line Feature Implementation**

Input: A set of stroke points \( P \).

Output: A distance measure

\[
\text{FitLine}(P) = \begin{cases} 
(1) & x_1 = \sum_{i=1}^{n} X(P_i) \\
(2) & y_1 = \sum_{i=1}^{n} Y(P_i) \\
(3) & x_2 = \sum_{i=1}^{n} X(P_i)^2 \\
(4) & y_2 = \sum_{i=1}^{n} Y(P_i)^2 \\
(5) & xy_1 = \sum_{i=1}^{n} X(P_i)Y(P_i) \\
(6) & x_3 = x_2 - x_1^2/n \\
(7) & y_3 = y_2 - y_1^2/n \\
(8) & xy_2 = xy_1 - (x_1y_1)/n \\
(9) & \text{rad} = \sqrt{(x_3 - y_3)^2 + 4xy_2^2} \\
(10) & \text{error} = \frac{(x_3 + y_3 - \text{rad})}{2} \\
(11) & \text{max} = \sqrt{\text{error}/n} \\
(12) & \text{if } x_3 > y_3 \\
(13) & a = -2xy_2 \\
(14) & b = x_3 - y_3 + \text{rad} \\
(15) & \text{else if } x_3 < y_3 \\
(16) & a = y_3 - x_3 + \text{rad} \\
(17) & b = -2xy_2 \\
(18) & \text{else} \\
(19) & \text{if } xy_2 = 0 \\
(20) & a = 0 - b = c = 0 \\
(21) & \text{error} = +\infty \\
(22) & \text{else} \\
(23) & a = 1 \\
(24) & b = 1 \\
(25) & \text{mag} = \sqrt{a^2 + b^2} \\
(26) & c = \frac{a - xy_2}{\text{mag}} \\
(27) & a = \frac{a}{\text{mag}} \\
(28) & b = \frac{b}{\text{mag}} \\
(29) & \text{min}_1 = +\infty \\
(30) & \text{max}_1 = -\infty \\
(31) & \text{for } i = 1 \text{ to } n \\
(32) & \text{err} = aX(P_i) + bY(P_i) + c \\
(33) & pX = X(P_i) - a \cdot \text{err} \\
(34) & pY = Y(P_i) - b \cdot \text{err} \\
(35) & \text{ploc} = b \cdot pX + b \cdot pY \\
(36) & \text{min}_4 = \min(\text{min}_4, \text{ploc}) \\
(37) & \text{max}_4 = \max(\text{max}_4, \text{ploc}) \\
(38) & \text{return } \frac{\text{ploc} - \text{min}_4}{\text{max}_4 - \text{min}_4} \\
\end{cases}
\]
Statistical Features (1)

- **Side ratios**
  - first and last point of strokes have variable locations with respect to the bounding box
  - **Approach**
    - take the x coordinates of the first and last point of a stroke
    - subtract them from the left side of the symbol’s bounding box (i.e., the bounding box’s leftmost x value)
    - divide by the bounding box width.

Statistical Features (2)

- **Top and Bottom ratios**
  - similar to side ratios except we are dealing with y coordinate
  - **Approach**
    - take y coordinate of the first and last point of a stroke
    - subtract from the top of the symbol’s bounding box (i.e., the bounding box’s topmost y value)
    - these values are divided by the bounding box height.
Statistical Features (3)

- **Point Histogram**
  - Distribution of point locations in stroke bounding box
  - Discrimination where point concentrations are high
  - Approach
    - Break up box into $n \times m$ grid
    - Count number of points in each sub box
    - Divide by total number of points

Statistical Features (4)

- **Angle Histogram**
  - Similar to point histogram except dealing with angles
  - Approach
    - Given $\vec{v}_j = p_j - p_{i-1}$ for $2 \leq i \leq n$ and $\vec{x} = (1,0)$
    - $\alpha_j = \arccos \left( \frac{\vec{x} \cdot \vec{v}_j}{\|\vec{v}_j\|} \right)$
    - Put angles into bins of $n$ degrees
The Rubine Feature Set (Rubine 1991)

- Part of Rubine’s gesture recognition system
  - we will see this next class
- Stroke
  - $P =$ total number of points
  - $p =$ middle point
  - first point ($x_0,y_0,t_0$)
  - last point ($x_{P-1},y_{P-1},t_{P-1}$)
  - compute $x_{\min}, y_{\min}, x_{\max}, y_{\max}$

Feature $f_1$

- Cosine of starting angle

$$f_1 = \cos(\alpha) = \frac{(x_2 - x_0)}{\sqrt{(x_2 - x_0)^2 + (y_2 - y_0)^2}}$$
Feature $f_2$

- Sine of starting angle

$$f_2 = \sin(\alpha) = \frac{(y_2 - y_0)}{\sqrt{(x_2 - x_0)^2 + (y_2 - y_0)^2}}$$

Feature $f_3$

- Length of diagonal of bounding box (gives an idea of the size of the bounding box)

$$f_3 = \sqrt{(x_{\text{max}} - x_{\text{min}})^2 + (y_{\text{max}} - y_{\text{min}})^2}$$
Feature $f_4$

- Angle of diagonal
- gives an idea of the shape of the bounding box (long, tall, square)

$$f_4 = \arctan\left(\frac{y_{\text{max}} - y_{\text{min}}}{x_{\text{max}} - x_{\text{min}}}\right)$$

Feature $f_5$

$$f_5 = \sqrt{(x_{p-1} - x_0)^2 + (y_{p-1} - y_0)^2}$$

- Distance from start to end of stroke
Feature $f_6$

- Cosine of ending angle

\[ f_6 = \cos(\beta) = \frac{(x_{p-1} - x_0)}{f_5} \]

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Feature $f_7$

- Sine of ending angle

\[ f_7 = \sin(\beta) = \frac{(x_{p-1} - x_0)}{f_5} \]
More Definitions (before we continue)

Let $\Delta x_p = x_{p+1} - x_p$ and $\Delta y_p = y_{p+1} - y_p$

Let $\theta_p = \arctan \frac{\Delta x_p \Delta y_{p-1} - \Delta x_{p-1} \Delta y_p}{\Delta x_p \Delta x_{p-1} + \Delta y_p \Delta y_{p-1}}$ Directional angle

Let $\Delta t_p = t_{p+1} - t_p$ Time delta

Feature $f_8$

- Total stroke length

$$f_8 = \sum_{p=0}^{P-2} \sqrt{\Delta x_p^2 + \Delta y_p^2}$$

- Diagram of stroke with points $(x_{p-1}, y_{p-1})$, $(x_p, y_p)$, and $(x_{p+1}, y_{p+1})$
**Feature f_9**

- Total rotation (from start to end point)
- (not the same as β-α – think of spirals)

\[ f_9 = \sum_{p=1}^{P-2} \theta_p \]

**Feature f_10**

- Absolute rotation
- How much does it move around

\[ f_{10} = \sum_{p=1}^{P-2} |\theta_p| \]
Feature $f_{11}$

- Rotation squared
- How smooth are the turns?
- Measure of sharpness

\[ f_{11} = \sum_{p=1}^{P-2} \theta_p^2 \]

Feature $f_{12}$

- The maximum speed reached (squared)

\[ f_{12} = \max_{p=0}^{P-2} \frac{\Delta x_p^2 + \Delta y_p^2}{\Delta t_p^2} \]
Feature $f_{13}$

- Total time of stroke

$$f_{13} = t_{P-1} - t_0$$