Features Extraction for Sketch-Based Recognition

Recall Pen-Based Interface Dataflow

- Raw Stroke Data
- Preprocessing
- Segmentation
- Sketch Understanding
- Ink Parsing
- Classification
- Feature Extraction and Analysis
- Make Inferences
Feature Extraction and Analysis

- What came first, the feature or the machine learning algorithm?
- Want to distinguish sketch components from one another
- Good features are critical
- Extract important information
  - geometrical, statistical, contextual
- Examples include
  - arc length, histograms, cusps, aspect ratio
  - self-intersections, stroke area, etc...

Finding Features

- Challenging problem
  - need fast algorithms for gathering information
  - features must be good discriminators
- Often trial and error
- Can be domain specific
Geometric Features (1)

- **Number of strokes**
  - if you know how many strokes a symbol has, you can break up your recognizer into pieces (i.e., recognizer for 1 stroke symbols, recognizer for 2 stroke symbols …)

- **Cusps**
  - smooth vs. jagged strokes
  - distance between cusps
    - useful for when cusps are close together/far apart

Geometric Features (2)

- **Aspect ratio (width / height)**
  - tall vs. flat

- **Self Intersections**
  - loops vs. no loops
  - strokes with write over
  - distance between self intersections also useful
  - use line segment intersection algorithm
Geometric Features (3)

- First and last distance
  - Strokes where first and last points are close together vs. far apart
  - simple computation \( \|p_n - p_1\| \)

- Arc length
  - many different symbols have varying arc lengths
  - simple computation as well –

\[
l = \sum_{i=2}^{n} \|p_i - p_{i-1}\|
\]

Geometric Features (4)

- Stroke area
  - area defined by the vectors created with the initial stroke point and consecutive stroke points.
  - good discriminator for straight vs. curved lines

Given \( \vec{u}_i = p_{i+1} - p_i \) and \( \vec{v}_i = p_{i+2} - p_i \)

\[
s_{area} = \sum_{i=1}^{n-2} \frac{1}{2} (\vec{u}_i \times \vec{v}_i) \cdot \text{sgn}(\vec{u}_i \times \vec{v}_i)
\]

where \( \vec{u}_i \times \vec{v}_i \) is a scalar
Geometric Features (5)

- **Fit line feature**
  - sophisticated approach to finding how close a stroke is to a straight line
  - finds a least-squares approximation to a line using principal components and then uses this approximation to find the distance of the projection of the stroke points onto the approximated line
  - outputs a value in [0, 1]
- What is another name for this approach?

### Fit Line Feature Implementation

**Input:** A set of stroke points \( P \).

**Output:** A distance measure

\[
\text{FitLine}(P) = \begin{align*}
1) & \quad x_1 = \sum_{i=1}^{n} X(P_i) \\
2) & \quad y_1 = \sum_{i=1}^{n} Y(P_i) \\
3) & \quad x_2 = \sum_{i=1}^{n} X(P_i)^2 \\
4) & \quad y_2 = \sum_{i=1}^{n} Y(P_i)^2 \\
5) & \quad xy_1 = \sum_{i=1}^{n} X(P_i)Y(P_i) \\
6) & \quad x_3 = x_2 - x_1^2/n \\
7) & \quad y_3 = y_2 - y_1^2/n \\
8) & \quad xy_2 = xy_1 - (x_1y_1)/n \\
9) & \quad \text{rad} = \sqrt{(x_3 - y_3)^2 + 4xy_2^2} \\
10) & \quad \text{error} = (x_3 + y_3 - \text{rad})/2 \\
11) & \quad \text{max} = \sqrt{\text{error}/n} \\
12) & \quad \text{if } x_3 > y_3 \\
13) & \quad a = -2xy_2 \\
14) & \quad b = x_3 - y_3 + \text{rad} \\
15) & \quad \text{else if } x_3 < y_3 \\
16) & \quad a = y_3 - x_3 + \text{rad} \\
17) & \quad b = -2xy_2 \\
18) & \quad \text{else} \\
19) & \quad \text{if } xy_2 = 0 \\
20) & \quad a = b = c = 0 \\
21) & \quad \text{error} = +\infty \\
22) & \quad \text{else} \\
23) & \quad a = -1 \\
24) & \quad b = -1 \\
25) & \quad \text{mag} = \sqrt{a^2 + b^2} \\
26) & \quad c = (a \cdot X(P_i) + b \cdot Y(P_i))/n \\
27) & \quad a = \text{mag} \\
28) & \quad b = \text{mag} \\
29) & \quad \text{min}_1 = +\infty \\
30) & \quad \text{max}_3 = -\infty \\
31) & \quad \text{for } i = 1 \text{ to } n \\
32) & \quad \text{err} = aX(P_i) + bY(P_i) + c \\
33) & \quad pX = X(P_i) - a \cdot \text{err} \\
34) & \quad pY = Y(P_i) - b \cdot \text{err} \\
35) & \quad p\text{loc} = -b \cdot pX + b \cdot pY \\
36) & \quad \text{min}_1 = \min(\text{min}_1, p\text{loc}) \\
37) & \quad \text{max}_1 = \max(\text{max}_1, p\text{loc}) \\
38) & \quad \text{return } \frac{\text{err} \cdot \text{rad}}{\text{max}_1 - \text{min}_1}
\end{align*}
\]
Statistical Features (1)

- Side ratios
  - first and last point of strokes have variable locations with respect to the bounding box
  - Approach
    - take the x coordinates of the first and last point of a stroke
    - subtract them from the left side of the symbol’s bounding box (i.e., the bounding box’s leftmost x value)
    - divide by the bounding box width.

Statistical Features (2)

- Top and Bottom ratios
  - similar to side ratios except we are dealing with y coordinate
  - approach
    - take y coordinate of the first and last point of a stroke
    - subtract from the top of the symbol’s bounding box (i.e., the bounding box’s topmost y value)
    - these values are divided by the bounding box height.
Statistical Features (3)

- **Point Histogram**
  - distribution of point locations in stroke bounding box
  - discrimination where point concentrations are high
  - approach
    - break up box into $n \times m$ grid
    - Count number of points in each sub box
    - divide by total number of points

Statistical Features (4)

- **Angle Histogram**
  - similar to point histogram except dealing with angles
  - Approach
    - Given $\vec{v}_j = p_i - p_{i-1}$ for $2 \leq i \leq n$ and $\vec{x} = (1,0)$
    - $\alpha_j = \arccos \left( \frac{\vec{x} \cdot \vec{v}_j}{\|\vec{v}_j\|} \right)$
    - put angles into bins of $n$ degrees
The Rubine Feature Set (Rubine 1991)

- Part of Rubine's gesture recognition system
  - we will see this next class
- Stroke
  - \( P \) = total number of points
  - \( p \) = middle point
  - first point \((x_0, y_0, t_0)\)
  - last point \((x_{P-1}, y_{P-1}, t_{P-1})\)
  - compute \( x_{\text{min}}, y_{\text{min}}, x_{\text{max}}, y_{\text{max}} \)

---

Feature \( f_1 \)

- Cosine of starting angle

\[
f_1 = \cos(\alpha) = \frac{(x_2 - x_0)}{\sqrt{(x_2 - x_0)^2 + (y_2 - y_0)^2}}
\]

---

Fall 2013  CAP 6105 – Pen-Based User Interfaces  ©Joseph J. LaViola Jr.
Feature $f_2$

- Sine of starting angle

$$f_2 = \sin(\alpha) = \frac{(y_2 - y_0)}{\sqrt{(x_2 - x_0)^2 + (y_2 - y_0)^2}}$$

Feature $f_3$

- Length of diagonal of bounding box (gives an idea of the size of the bounding box)

$$f_3 = \sqrt{(x_{\text{max}} - x_{\text{min}})^2 + (y_{\text{max}} - y_{\text{min}})^2}$$
Feature \( f_4 \)

- Angle of diagonal
- Gives an idea of the shape of the bounding box (long, tall, square)

\[
f_4 = \arctan \left( \frac{y_{\text{max}} - y_{\text{min}}}{x_{\text{max}} - x_{\text{min}}} \right)
\]

Feature \( f_5 \)

- Distance from start to end of stroke

\[
f_5 = \sqrt{(x_{p-1} - x_0)^2 + (y_{p-1} - y_0)^2}
\]
Feature $f_6$

- Cosine of ending angle

$$ f_6 = \cos(\beta) = \frac{(x_{p-1} - x_0)}{f_5} $$

Feature $f_7$

- Sine of ending angle

$$ f_7 = \sin(\beta) = \frac{(x_{p-1} - x_0)}{f_5} $$
More Definitions (before we continue)

Let $\Delta x_p = x_{p+1} - x_p$ and $\Delta y_p = y_{p+1} - y_p$

Let $\theta_p = \arctan \frac{\Delta x_p \Delta y_{p-1} - \Delta x_{p-1} \Delta y_p}{\Delta x_p \Delta x_{p-1} + \Delta y_p \Delta y_{p-1}}$ Directional angle

Let $\Delta t_p = t_{p+1} - t_p$ Time delta

Feature $f_8$

- Total stroke length

$$f_8 = \sum_{p=0}^{P-2} \sqrt{\Delta x_p^2 + \Delta y_p^2}$$
Feature \( f_9 \)

- Total rotation (from start to end point)
- (not the same as \( \beta - \alpha \) – think of spirals)

\[
f_9 = \sum_{p=1}^{P-2} \theta_p
\]

Feature \( f_{10} \)

- Absolute rotation
- How much does it move around

\[
f_{10} = \sum_{p=1}^{P-2} |\theta_p|
\]
Feature $f_{11}$

- Rotation squared
- How smooth are the turns?
- Measure of sharpness

$$f_{11} = \sum_{p=1}^{P-2} \theta_p^2$$

Feature $f_{12}$

- The maximum speed reached (squared)

$$f_{12} = \max_{p=0}^{P-2} \frac{\Delta x_p^2 + \Delta y_p^2}{\Delta t_p^2}$$
Feature $f_{13}$

- Total time of stroke

$$f_{13} = t_{P-1} - t_0$$

Readings