Ink Parsing in Sketch-Based Interfaces

Recall Pen-Based Interface Dataflow

- Raw Stroke Data
- Preprocessing
- Segmentation
- Sketch Understanding
- Ink Parsing
- Classification
- Make Inferences
- Feature Extraction and Analysis
Sketch Parsing

- Often recognition of strokes is insufficient
  - except for gestures
- Require an understanding of spatial relationships
  - good examples are mathematical expressions
- Higher level classifications
  - is it a word or a drawing?

Approaches to Sketch Parsing

- Top down vs. bottom up
- Focus on mathematical expressions
  - 2D (coordinate) grammars
  - graph rewriting
    - useful for other types of parsing as well (diagrams, tables, lists, etc…)
  - projection profile cutting
  - procedurally coded syntax rules
  - stochastic grammars
- Other parsing approaches
  - conditional random fields
  - statistical visual languages
  - many others
2D Grammars

- Grammar + spatial relationship rules
  - useful if a well defined syntax exists
  - looks for key symbols
- One Approach – Box Grammar
  - divide input into distinct areas based on symbol found

Graph Rewriting

- Expressions represented as nodes and arcs
- Rewrite rules applied to graph to reduce it progressively
  - rules are also subgraphs
  - graph reduced to single node representing expression
Graph Rewriting Example (Blostein and Grbavec 1996)

- **Build**
  - add edges between symbols (above, below, left, superscript, subscript)
- **Constrain**
  - Apply knowledge of notational conventions
    - remove contradictory associations
    - disambiguate horizontal lines
    - disambiguate dots
    - disambiguate diagonal associations
- **Rank**
  - Use information about operator precedence to group symbols into subexpressions
- **Incorporate**
  - Interpret subexpressions

Projection Profile Cutting

- Used primarily in document analysis
- Uses horizontal and vertical projections of expression onto x and y axis
  - subdivides expression recursively
- Problem with expressions where symbols are close together (no white space)
Procedurally Coded Syntax Rules

- Observations about domain coded programmatically
  - similar to rule based approach for recognition
- Sample rule for horizontal line

A length threshold of 20 pixels is used to classify a horizontal line as a short or long bar.
If it is a long bar and has symbols above and below, it is treated as a division.
If there are no symbols above, it is treated as a boolean negation.
If a short bar has no symbols above or below, it is treated as minus sign.
If it has symbols above or below, the combination symbols such as =, ≤, and ≥ are formed.

Stochastic Grammars

- Used to deal with noisy data and spatial ambiguities
- Probabilities associated with each production rule
- For any sequence in a given parse – probability can be calculated
- Requires training
MathPad\(^2\) Parsing Approach

- Uses 2D coordinate grammar approach with some syntax rules
- Basic approach
  - preprocessing step (for functions)
  - sort list of symbols
  - parse functions – use grammar
  - process functions – handle spatial relationship testing
    - intermixed with parse functions

Grammar (1)

```
<math_formula> ::= <equation> | <expression>
<equation> ::= <expression> <relational_op> <expression> | <expression> \(=\) <cond_expression>
<relational_op> ::= \(=\) | \(<\) | \(>\) | \(<=\) | \(>=\)
<cond_expression> ::= \(\{\) <cond_statements> \(\}\)
<cond_statements> ::= \(\{\) <expression> \(\};\) <logic_expression> \("\mathbf{else}\" <expression> \("\mathbf{else}\" <logic_expression> \} <expression> \("\mathbf{if}\"
<logic_expression> ::= <equation> <logical_op> <logic_expression> | <equation>
<logical_op> ::= \("\mathbf{and}\") | \("\mathbf{or}\")
<expression> ::= <term> <"\cdot"<expression> | <term> <"\cdot"<expression> | <term> <"\cdot"<expression> | <term>
<term> ::= <factor> <"\cdot"<term> | <"\cdot"<expression> | <factor>
<factor> ::= <sub_expression> <"\prime\prime"<factor> | <sub_expression>
<sub_expression> ::= <integral> | <derivative> | <summation> | <function> | <terminal>
```
### Parse functions

- High level parse
- Expression parse
- Sub-expression parse
- Symbol specific parsing
  - square root parse
  - integration parse
  - summation parse
  - fraction parse
- Factor parse
- Term parse
Process functions

- Provide parse functions important info
- Deal with spatial relationships
  - implicit operators
  - fractions and square roots
  - summations, derivatives, integrals
  - Conditionals

\[
x(t+h) = \begin{cases} \ell - r : x(t) > (\ell - r) \\ r : x(t) < r \\ x(t) + vh : \text{else} \end{cases}
\]

Reducing parsing decisions

- Use application to reduce decisions
- Implicit operators (no numbers have subscripts)
- Correct trig functions 5in -> sin
- Functions of time f(+) -> f(t)
Readings