Features Extraction for Sketch-Based Recognition

Lecture #8: Feature Extraction
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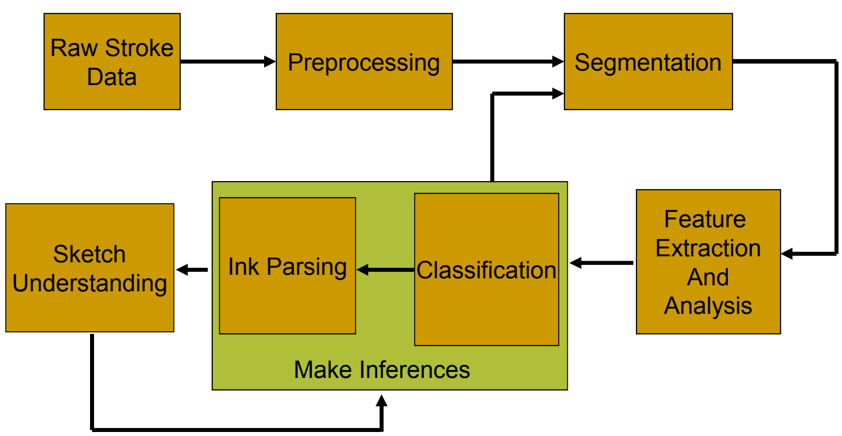
Fall 2010

Recall Pen-Based Interface Dataflow



Feature Extraction and Analysis

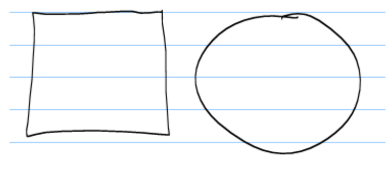
- What came first, the feature or the machine learning algorithm?
- Want to distinguish sketch components from one another
- Good features are critical
- Extract important information
  - geometrical, statistical, contextual
- Examples include
  - arc length, histograms, cusps, aspect ratio
  - self-intersections, stroke area, etc…

Finding Features

- Challenging problem
  - need fast algorithms for gathering information
  - features must be good discriminators
- Often trial and error
- Can be domain specific

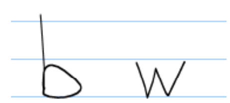
Geometric Features (1)

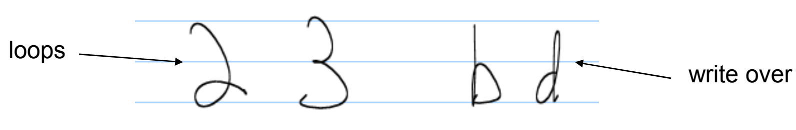
- Number of strokes
  - if you know how many strokes a symbol has, you can break up your recognizer into pieces (i.e., recognizer for 1 stroke symbols, recognizer for 2 stroke symbols …)
- Cusps
  - smooth vs. jagged strokes
  - distance between cusps
    - useful for when cusps are close together/far apart



Geometric Features (2)

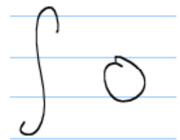
- Aspect ratio (width / height)
  - tall vs. flat
- Self Intersections
  - loops vs. no loops
  - strokes with write over
  - distance between self intersections also useful
  - use line segment intersection algorithm





Geometric Features (3)

- First and last distance
  - Strokes where first and last points are close together vs. far apart
  - simple computation – $\|p_n - p_1\|$



- Arc length
  - many different symbols have varying arc lengths
  - simple computation as well –

$$l = \sum_{i=2}^{n} \|p_i - p_{i-1}\|$$

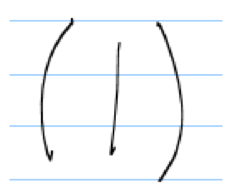
Geometric Features (4)

- Stroke area
  - area defined by the vectors created with the initial stroke point and consecutive stroke points.
  - good discriminator for straight vs. curved lines

$$\text{Given } \vec{u}_i = p_{i+1} - p_1 \text{ and } \vec{v}_i = p_{i+2} - p_1$$

$$s_{area} = \sum_{i=1}^{n-2} \frac{1}{2}\left(\vec{u}_i \times \vec{v}_i\right) \cdot \operatorname{sgn}\left(\vec{u}_i \times \vec{v}_i\right)$$

$$\text{where } \vec{u}_i \times \vec{v}_i \text{ is a scalar}$$



Geometric Features (5)

- Fit line feature
  - sophisticated approach to finding how close a stroke is to a straight line
  - finds a least-squares approximation to a line using principal components and then uses this approximation to find the distance of the projection of the stroke points onto the approximated line
  - outputs a value in [0,1]
- What is another name for this approach?

Fit Line Feature Implementation

Input: A set of stroke points P.
Output: A distance measure
FitLine(P)

$x_1 \leftarrow \sum_{i=1}^{n} X(P_i)$
$y_1 \leftarrow \sum_{i=1}^{n} Y(P_i)$
$x_2 \leftarrow \sum_{i=1}^{n} X(P_i)^2$
$y_2 \leftarrow \sum_{i=1}^{n} Y(P_i)^2$
$xy_1 \leftarrow \sum_{i=1}^{n} X(P_i)Y(P_i)$
$x_3 \leftarrow x_2 - x_1^2/n$
$y_3 \leftarrow y_2 - y_1^2/n$
$xy_2 \leftarrow xy_1 - (x_1 y_1)/n$
$rad \leftarrow \sqrt{(x_3 - y_3)^2 + 4xy_2^2}$
$error \leftarrow (x_3 + y_3 - rad)/2$
$rms \leftarrow \sqrt{error/n}$
if $x_3 > y_3$
  $a \leftarrow -2xy_2$
  $b \leftarrow x_3 - y_3 + rad$
else if $x_3 < y_3$
  $a \leftarrow y_3 - x_3 + rad$
  $b \leftarrow -2xy_2$
else
  if $xy_2 = 0$
    $a \leftarrow b \leftarrow c \leftarrow 0$
    $error \leftarrow +\infty$
  else
    $a \leftarrow 1$
    $b \leftarrow -1$
$mag \leftarrow \sqrt{a^2 + b^2}$
$c \leftarrow \frac{(-ax_1 - by_1)/n}{mag}$
$a \leftarrow \frac{a}{mag}$
$b \leftarrow \frac{b}{mag}$
$min_1 \leftarrow +\infty$
$max_1 \leftarrow -\infty$
for i=1 **to** n
  $err \leftarrow aX(P_i) + bY(P_i) + c$
  $pX \leftarrow X(P_i) - a \cdot err$
  $pY \leftarrow Y(P_i) - b \cdot err$
  $ploc \leftarrow -b \cdot pX + b \cdot pY$
  $min_1 \leftarrow \min(min_1, ploc)$
  $max_1 \leftarrow \max(max_1, ploc)$
return $\frac{100 \cdot rms}{max - min}$

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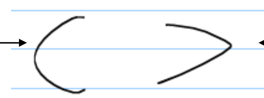
Statistical Features (1)

- Side ratios
  - first and last point of strokes have variable locations with respect to the bounding box
  - Approach
    - take the x coordinates of the first and last point of a stroke
    - subtract them from the left side of the symbol's bounding box (i.e., the bounding box's leftmost x value)
    - divide by the bounding box width.

1st and last point on right side of bbox

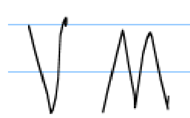


1st and last point on left side of bbox

Statistical Features (2)

- Top and Bottom ratios
  - similar to side ratios except we are dealing with *y* coordinate
  - approach
    - take y coordinate of the first and last point of a stroke
    - subtract from the top of the symbol's bounding box (i.e., the bounding box's topmost y value)
    - these values are divided by the bounding box height.

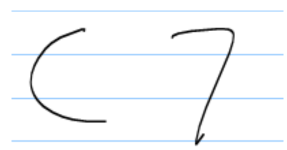
1st and last points close to top of bbox



1st and last points are at the bottom of bbox

Statistical Features (3)

- Point Histogram
  - distribution of point locations in stroke bounding box
  - discrimination where point concentrations are high
  - approach
    - break up box into *n x m* grid
    - Count number of points in each sub box
    - divide by total number of points



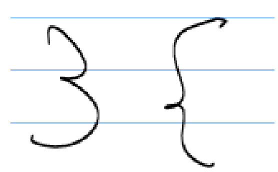
Statistical Features (4)

- Angle Histogram
  - similar to point histogram except dealing with angles
  - Approach

$$\text{Given } \vec{\mathrm{v}}_{\mathrm{j}} = p_i - p_{i-1} \text{ for } 2 \le i \le n \text{ and } \vec{x} = (1,0)$$

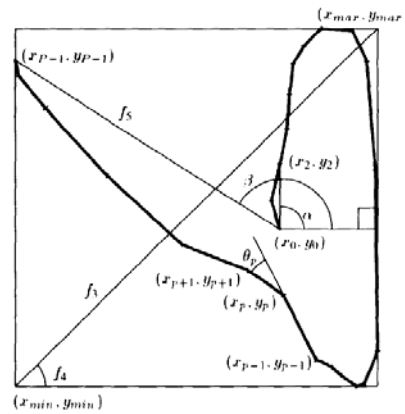
$$\alpha_j = \arccos\left(\vec{x} \cdot \frac{\vec{\mathrm{v}}_{\mathrm{j}}}{\left\| \vec{\mathrm{v}}_{\mathrm{j}} \right\|} \right)$$

    - put angles into bins of *n* degrees



The Rubine Feature Set (Rubine 1991)

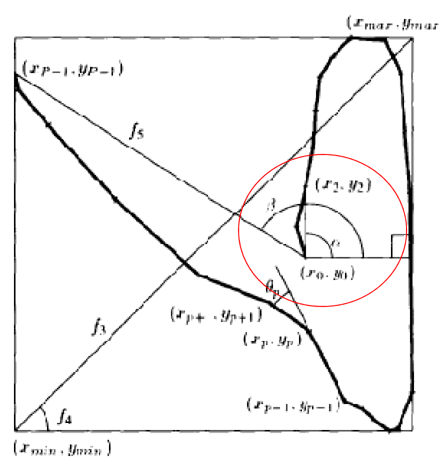
- Part of Rubine’s gesture recognition system
  - we will see this next class
- Stroke
  - P = total number of points
  - p = middle point
  - first point (x_0, y_0, t_0)
  - last point $(x_{P-1}, y_{P-1}, t_{P-1})$
  - compute x_{min}, y_{min}, x_{max}, y_{max}



Feature f_1

- Cosine of starting angle

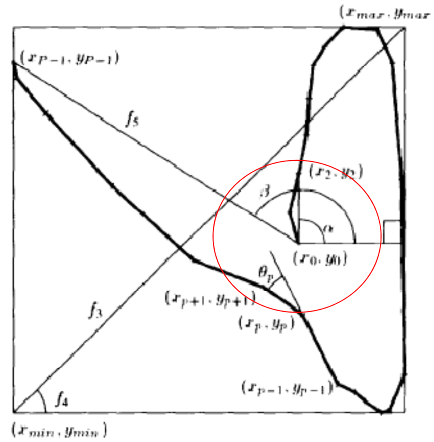
$$f_1 = \cos(\alpha) = \frac{(x_2 - x_0)}{\sqrt{(x_2 - x_0)^2 + (y_2 - y_0)^2}}$$



Feature f_2

- Sine of starting angle

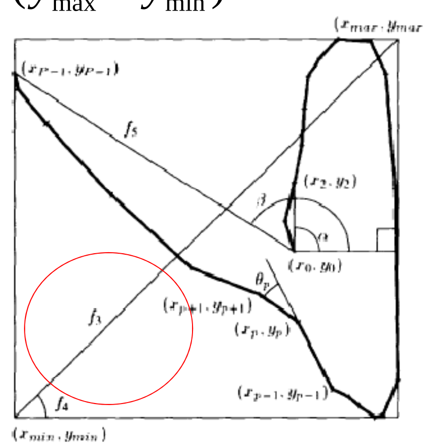
$$f_2 = \sin(\alpha) = \frac{(y_2 - y_0)}{\sqrt{(x_2 - x_0)^2 + (y_2 - y_0)^2}}$$



Feature f_3

$$f_3 = \sqrt{(x_{\max} - x_{\min})^2 + (y_{\max} - y_{\min})^2}$$

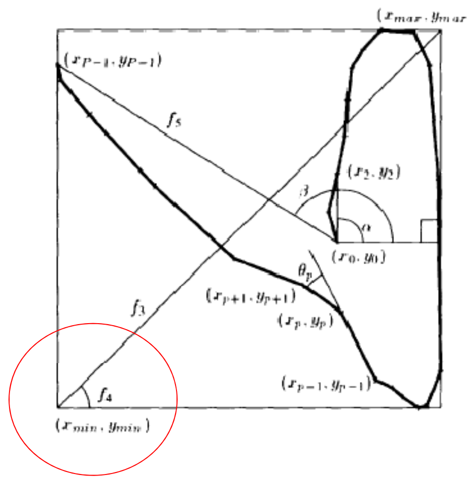
- Length of diagonal of bounding box (gives an idea of the size of the bounding box)



Feature f_4

- Angle of diagonal
- gives an idea of the shape of the bounding box (long, tall, square)

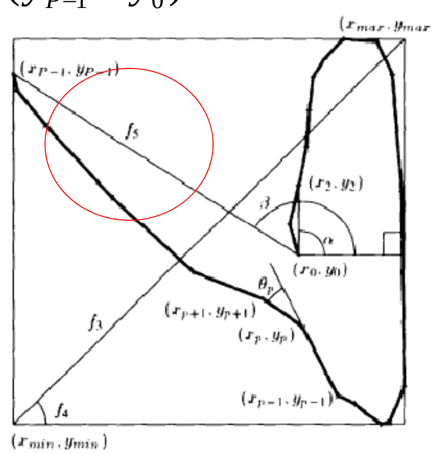
$$f_4 = \arctan\left(\frac{y_{\max} - y_{\min}}{x_{\max} - x_{\min}}\right)$$



Feature f_5

$$f_5 = \sqrt{(x_{P-1} - x_0)^2 + (y_{P-1} - y_0)^2}$$

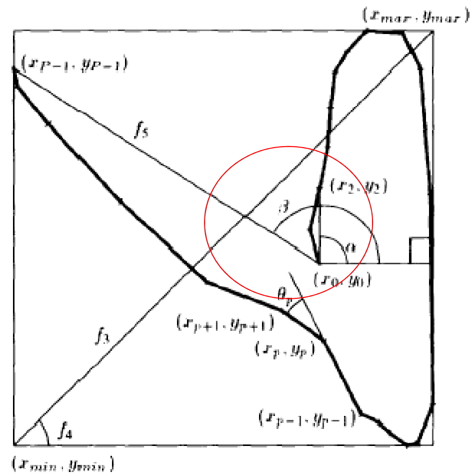
- Distance from start to end of stroke



Feature f_6

- Cosine of ending angle

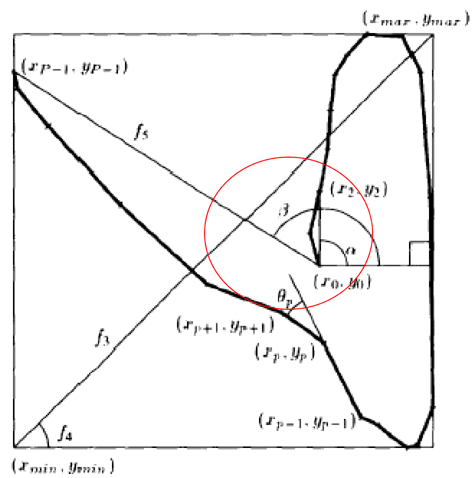
$$f_6 = \cos(\beta) = \frac{(x_{P-1} - x_0)}{f_5}$$



Feature f_7

- Sine of ending angle

$$f_7 = \sin(\beta) = \frac{(x_{P-1} - x_0)}{f_5}$$



More Definitions (before we continue)

$$\text{Let } \Delta x_p = x_{p+1} - x_p \text{ and } \Delta y_p = y_{p+1} - y_p$$

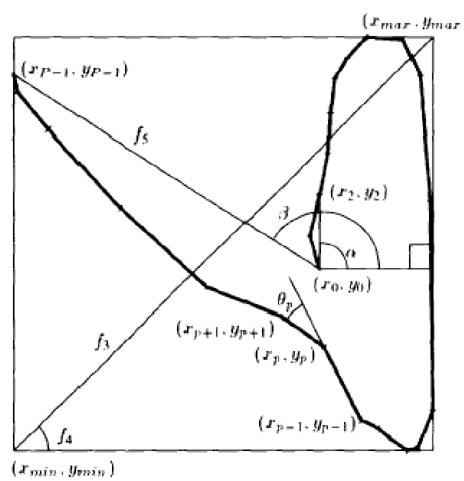
$$\text{Let } \theta_p = \arctan \frac{\Delta x_p \Delta y_{p-1} - \Delta x_{p-1} \Delta y_p}{\Delta x_p \Delta x_{p-1} + \Delta y_p \Delta y_{p-1}}$$ Directional angle

$$\text{Let } \Delta t_p = t_{p+1} - t_p$$ Time delta

Feature f_8

- Total stroke length

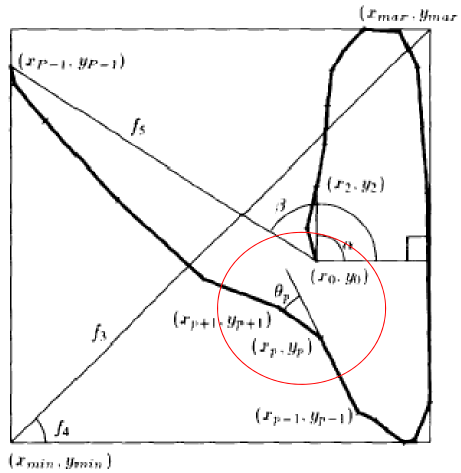
$$f_8 = \sum_{p=0}^{P-2} \sqrt{\Delta x_p^2 + \Delta y_p^2}$$



Feature f_9

- Total rotation (from start to end point)
- (not the same as β-α – think of spirals)

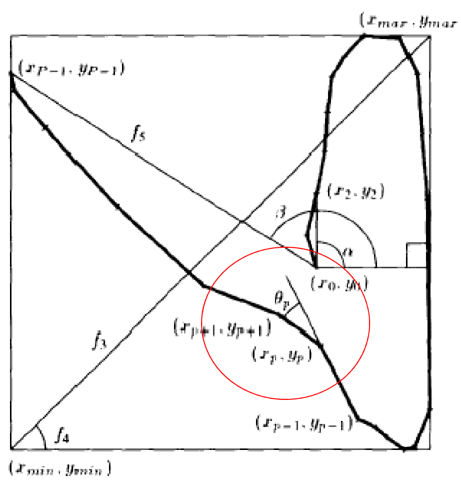
$$f_9 = \sum_{p=1}^{P-2} \theta_p$$



Feature f_{10}

- Absolute rotation
- How much does it move around

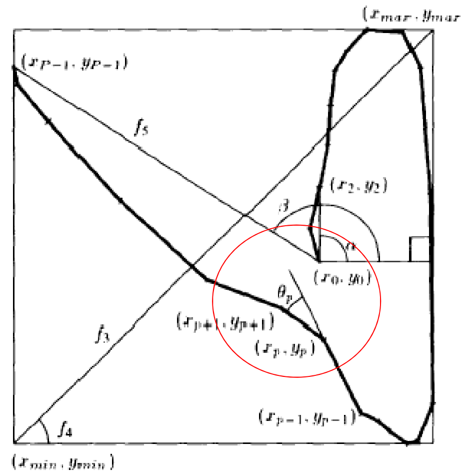
$$f_{10} = \sum_{p=1}^{P-2} |\theta_p|$$



Feature f_{11}

- Rotation squared
- How smooth are the turns?
- Measure of sharpness

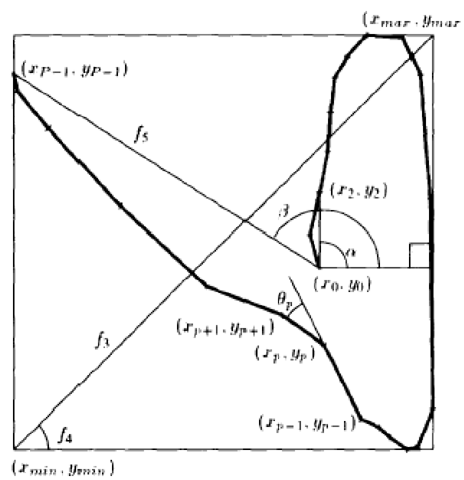
$$f_{11} = \sum_{p=1}^{P-2} \theta_p^2$$



Feature f_{12}

- The maximum speed reached (squared)

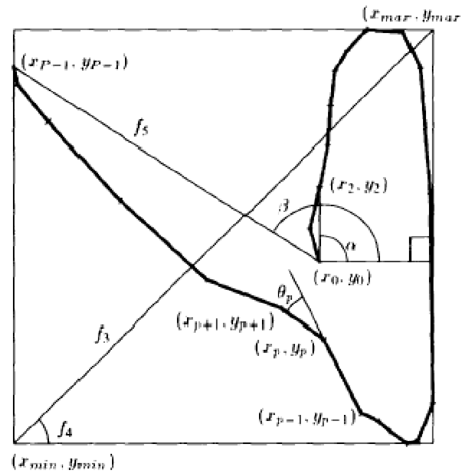
$$f_{12} = \max_{p=0}^{P-2} \frac{\Delta x_p^2 + \Delta y_p^2}{\Delta t_p^2}$$



Feature f_{13}

- Total time of stroke

$$f_{13} = t_{P-1} - t_0$$



Next Class

- Start discussing machine learning algorithms
  - linear classifiers (e.g., Rubine)
  - template matching
  - SVM
  - AdaBoost
  - etc…