CAP 5415 Computer Vision
Fall 2005

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Anandan's Approach
Project Assignment

$$I_x u + I_y v = -I_t \quad \Rightarrow \quad \begin{bmatrix} I_x & I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = -I_t$$

$$u = a_1 x + a_2 y + b_1$$
$$v = a_3 x + a_4 y + b_2$$

$$\Rightarrow \quad \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} x & y & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x & y & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ b_1 \\ a_3 \\ a_4 \\ b_2 \end{bmatrix}$$

$$\begin{bmatrix} I_x & I_y \end{bmatrix} \begin{bmatrix} x & y & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x & y & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ b_1 \\ a_3 \\ a_4 \\ b_2 \end{bmatrix} = -I_t$$

Anandan's Approach
Project Assignment

$$\sum_{\text{all pixels}} \mathbf{X}^T \Delta I \Delta I^T \mathbf{X}\mathbf{a} = -\sum_{\text{all pixels}} \mathbf{X}^T \Delta I \; I_t$$

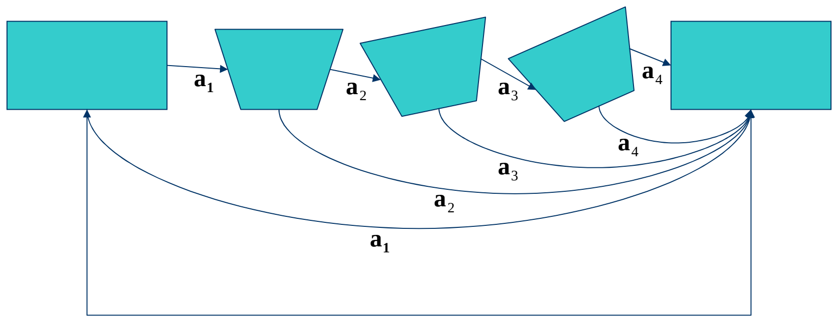
$$\left(\sum_{\text{all pixels}} \begin{bmatrix} x^2 I_x^2 & xyI_x^2 & xI_x^2 & x^2 I_x I_y & xyI_x I_y & xI_x I_y \\ xyI_x^2 & y^2 I_x^2 & yI_x^2 & xyI_x I_y & y^2 I_x I_y & yI_x I_y \\ xI_x^2 & yI_x^2 & I_x^2 & xI_x I_y & yI_x I_y & I_x I_y \\ x^2 I_x I_y & xyI_x I_y & xI_x I_y & x^2 I_y^2 & xyI_x^2 & xI_x^2 \\ xyI_x I_y & y^2 I_x I_y & yI_x I_y & xyI_x^2 & y^2 I_y^2 & yI_y^2 \\ xI_x I_y & yI_x I_y & I_x I_y & xI_x^2 & yI_y^2 & I_y^2 \end{bmatrix} \right) \begin{bmatrix} a_1 \\ a_2 \\ b_1 \\ a_3 \\ a_4 \\ b_2 \end{bmatrix} = \sum_{\text{all pixels}} \begin{bmatrix} -xI_x I_t \\ -yI_x I_t \\ -I_x I_t \\ -xI_y I_t \\ -yI_y I_t \\ -I_y I_t \end{bmatrix}$$

Anandan's Approach
Project Assignment HINTS

- Normalize the image size between [-0.7,+0.7] in both directions
  - To provide equal weighting for all the pixels
- Compute the image derivatives in both images using the following filter pairs
  - Multiply the derivatives with your normalization factor

Derivative in x		Derivative in y		Derivative in t	
$\begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}$	$\begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$	$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$
1st image	2nd image	1st image	2nd image	1st image	2nd image

Anandan’s Approach
Project Assignment



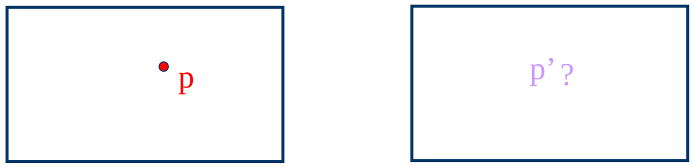
a Concatenation of **a**$_i$

$$A = A_2 A_1 \qquad B = A_2 B_1 + B_2$$

Anandan’s Approach
Project Assignment HINTS

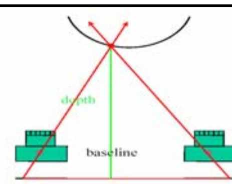
- Pyramid implementation
  1. Start from the smallest resolution
  2. Iteratively compute affine parameters (5 iteration)
  3. Go to a bigger scale
     - Keep A matrix as it is multiply B vector by 2
  4. Repeat steps 2 and 3 until all the pyramid levels are visited

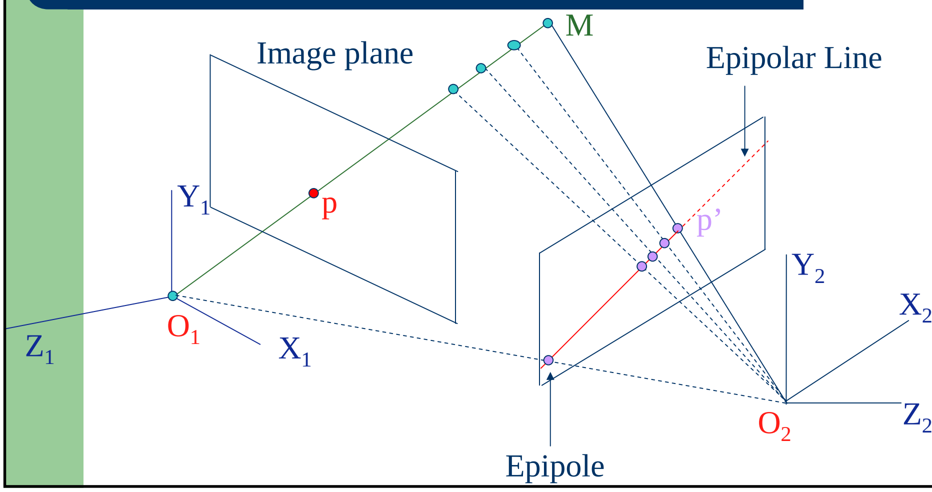
Stereo Constraints



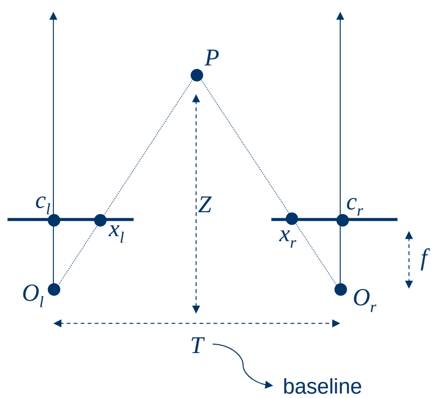
Given p in left image, where can the corresponding point p’ in right image be?

Stereo Constraints
Epipolar Geometry





Basic Stereo



$$Z = \frac{fT}{\boxed{x_1 - x_2}}$$

disparity

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Recap
Stereo Approaches

- Token Based Stereo
  - Find tokens (Corners, etc.) in both images
  - Find correspondences between tokens
  - Compute depth surface using disparity
- Computing Correspondence
  - Marr-Poggio (you don’t need to know)
  - Correlation based stereo methods
  - Barnard’s algorithm

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Recap
Correlation Based Stereo Methods

- Find corresponding tokens using correlation

SSD $= \sum\sum (I_{t+1} - I_t)^2$

NC $= \dfrac{\sum\sum (I_{t+1} . I_t)}{\sqrt{\sum\sum I_t . I_t}}$

AD $= \sum\sum |(I_{t+1} - I_t)|$

MC $= \dfrac{1}{64\sigma_{t+1}\sigma_t} \sum\sum (I_{t+1} - \mu_{t+1})(I_t - \mu_t)$

CC $= \sum\sum I_{t+1} I_t$

- Given correspondences compute depth

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Recap
Barnard's Stereo Method

- Define an energy based on intensity similarity

$$\mathrm{E} = \sum_{i=-1}^{1}\sum_{j=-1}^{1} \left\| I_{left}(x+i, y+j) - I_{right}(x+i+D_x(x,y), y+j+D_x(x,y)) \right\| + \lambda \left\| \nabla D(x,y) \right\|$$

$$\nabla D(x,y) = \sum_{i=-1}^{1}\sum_{j=-1}^{1} \left| D(x+i, y+j) - D(x,y) \right|$$

- Minimize energy using simulated annealing

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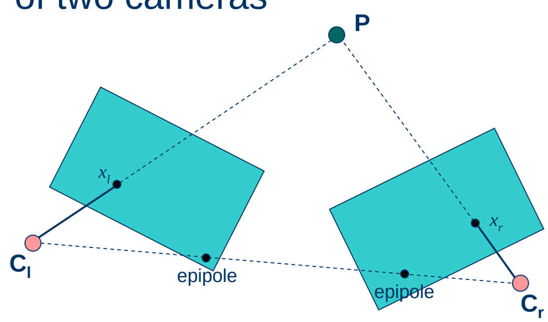
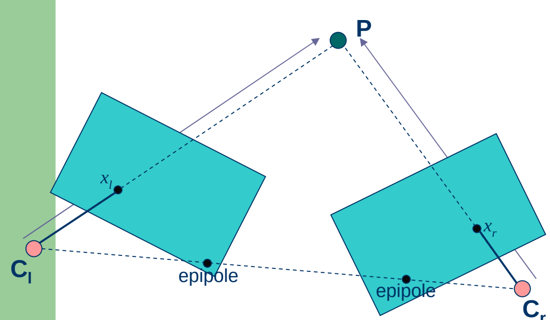
Fundamental Matrix

- Defines relation between two image planes of two cameras



Basics



$$T = C_l C_r$$

$$P_l = PC_l$$

$$\boxed{P_r = R(P_l - T)}$$

- Co-planarity condition
  - 3 vectors on a plane

    $$A^T(B \times C) = 0$$

  - Let's define for epipolar plane

    $$\boxed{(P_l - T)^T}(T \times P_l) = 0$$

    $$\boxed{(R^T P_r)^T}(T \times P_l) = 0$$

    $$P_r^T RSP_l = 0$$

Essential Matrix

- Related to camera extrinsic parameters
  - Rotation and translation

$$P_r^T RSP_l = 0$$

$$E = RS$$

- Captures relation between to camera coordinates

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How do we relate image coordinates?

- From camera coordinates to image coordinates.
  - Intrinsic camera parameters (in homogenous coordinates)

$$x_r^T M_r^{-T} RSM_t^{-1} x_l = 0$$

$$E = RS$$

$$\left.\begin{aligned} F &= M_r^{-T} RSM_t^{-1} \\ F &= M_r^{-T} EM_t^{-1} \end{aligned}\right\} \text{Fundamental matrix}$$

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Fundamental Matrix

- Relates left image coordinates to right image coordinates.
- Includes all camera parameters
- 3x3 matrix with 7 degrees of freedom
  - 4 for epipoles (x,y) and 3 for point projection
- Rank 2 (due to rank deficient S)

$$[x \quad y \quad 1] F \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = 0$$

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$$[x \quad y \quad 1] F \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = 0$$

Computing Fundamental Matrix

- Eight point algorithm
  - Most common approach
  - Requires 8 corresponding points in both images
- Write unknown fundamental matrix parameters $(f_1..f_9)$ into vector and the rest into equation (observation) matrix **O**.

$$\mathbf{O}.f = 0$$

- Compute least squares solution

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Computing Fundamental Matrix

- Impose rank constraint using SVD
- Constrain solution of f to $|f|=1$