CAP 5415 Computer Vision
Fall 2005

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Recap
3D Motion

- Displacement model

$$X' = X - \alpha Y + \beta Z + T_X$$
$$Y' = \alpha X + Y - \gamma Z + T_Y$$
$$Z' = -\beta X + \gamma Y + Z + T_Z$$

- Velocity model

$$\dot{X} = \Omega_2 Z - \Omega_3 Y + V_1$$
$$\dot{Y} = \Omega_3 X - \Omega_1 Z + V_2$$
$$\dot{Z} = \Omega_1 Y - \Omega_2 X + V_3$$

- 2D optical flow

orthographic projection

$$u = \Omega_2 Z - \Omega_3 y + V_1$$
$$v = \Omega_3 x - \Omega_1 Z + V_2$$

perspective projection

$$u = f(\frac{V_1}{Z} + \Omega_2) - \frac{V_3}{Z}x - \Omega_3 y - \frac{\Omega_1}{f}xy + \frac{\Omega_2}{f}x^2$$
$$v = f(\frac{V_2}{Z} - \Omega_1) + \Omega_3 x - \frac{V_3}{Z}y + \frac{\Omega_2}{f}xy - \frac{\Omega_1}{f}y^2$$

$$x' = x - \alpha y + \beta Z + T_X$$
$$y' = \alpha x + y - \gamma Z + T_Y$$

Recap
SFM (Orthographic Displacement)

- Two step simple approach
  - Assume depth Z is known compute motion parameters α, β, γ, T_x, T_y
  - Using new motion parameters refine depth Z

$$\begin{bmatrix} x' - x \\ y' - y \end{bmatrix} = \begin{bmatrix} -y & Z & 0 & 1 & 0 \\ x & 0 & -Z & 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \\ \gamma \\ T_x \\ T_y \end{bmatrix}$$

Step 1

$$\begin{bmatrix} \beta \\ -\gamma \end{bmatrix} [Z] = \begin{bmatrix} x' - x - \alpha y - T_x \\ y' - y - \alpha x - T_y \end{bmatrix}$$

Step 2

$$u = -(\frac{V_1}{Z} + \Omega_2) + \frac{V_3}{Z} x + \Omega_3 y + \Omega_1 xy - \Omega_2 x^2$$
$$v = -(\frac{V_2}{Z} - \Omega_1) - \Omega_3 x + \frac{V_3}{Z} y - \Omega_2 xy + \Omega_1 y^2$$

Recap
SFM (Perspective Motion)

- Find translation using search.
- Find rotation using least squares fit.
- Find Depth.

$$\Theta = \mathbf{C}(\mathbf{V})\mathbf{q} \qquad E(\mathbf{V},\mathbf{q}) = \|\Theta - \mathbf{C}(\mathbf{V})\mathbf{q}\|^2$$

$\mathbf{q}$ → 1/Z and rotations

Θ → optical flow

$\mathbf{C}(\mathbf{V})$ → matrix composed of translation and observations

$$E(\mathbf{V},\mathbf{q}) = \|\Theta - \mathbf{C}(\mathbf{V})\mathbf{q}\|^2$$

Recap
SFM (Perspective Motion)

- Translation search
  - Find derivative wrt. **q** and replace in *E*
  - Search for possible translations minimizing *E*
- Find Rotation
  - Define a vector perpendicular to ***p*A**
  - Least squares solution to Ω

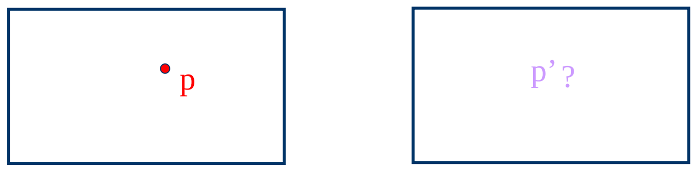
$$\overbrace{d^T(x,y,V)\Theta(\mathbf{x},\mathbf{y})}^{D} = \overbrace{d^T(x,y,V)\mathbf{B}(x,y)}^{E}\Omega$$

- Find depth by putting all values in

$$u = f(\frac{V_1}{Z} + \Omega_2) - \frac{V_3}{Z}x - \Omega_3 y - \frac{\Omega_1}{f}xy + \frac{\Omega_2}{f}x^2 \qquad v = f(\frac{V_2}{Z} - \Omega_1) + \Omega_3 x - \frac{V_3}{Z}y + \frac{\Omega_2}{f}xy - \frac{\Omega_1}{f}y^2$$

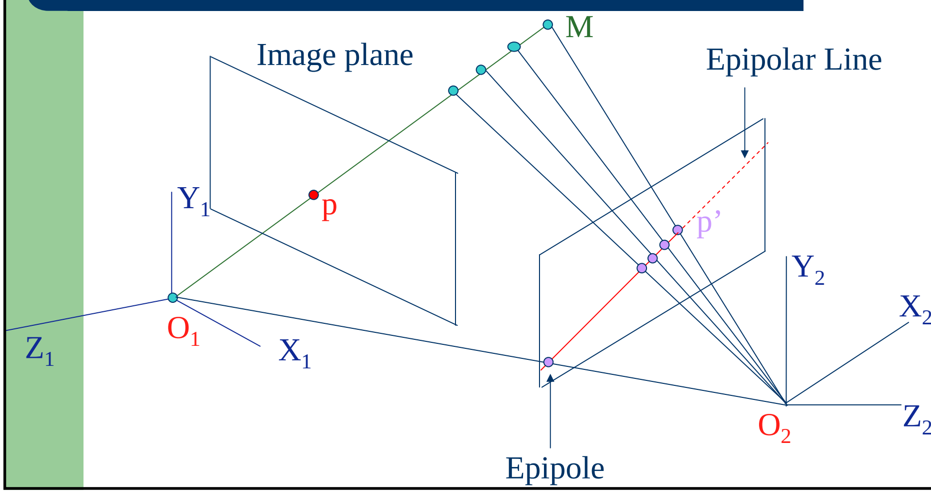
Multi-View Geometry

Stereo Constraints



Given p in the left image, where can the corresponding point p' be in the right image?

Stereo Constraints
Epipolar Geometry



Stereopsis

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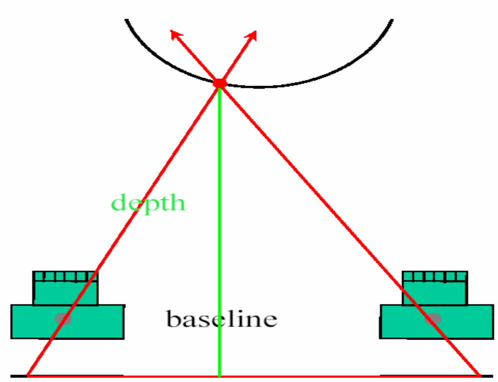
Definition



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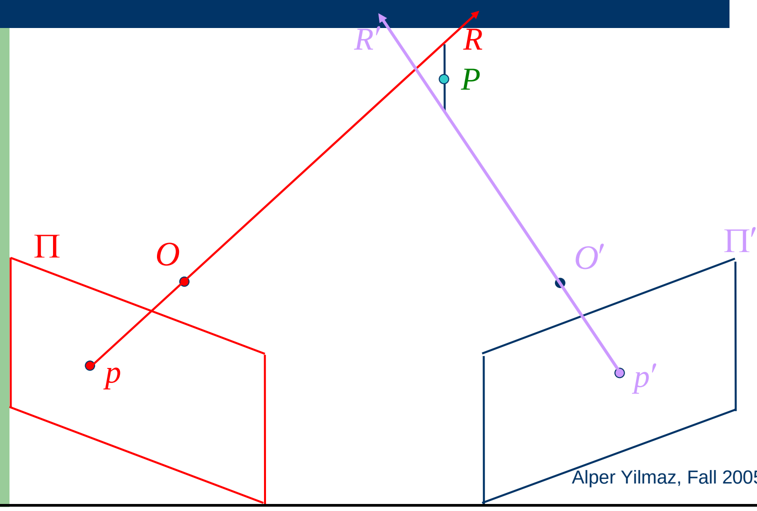
Definition



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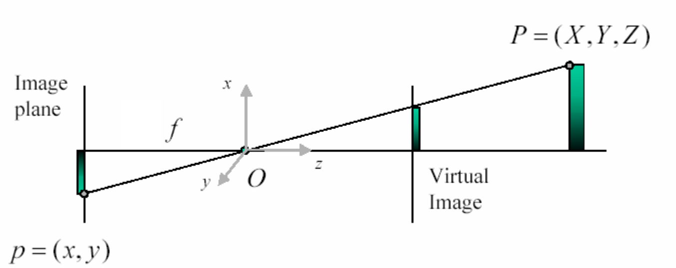
Pin Hole Camera Model



$$x = f\frac{X}{Z}$$

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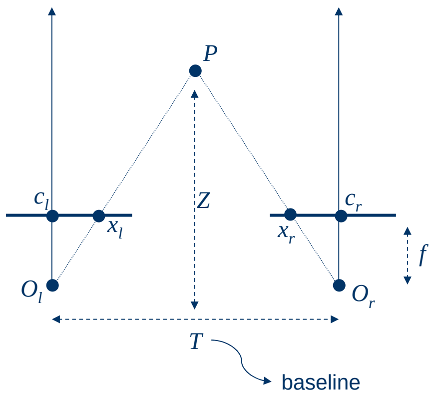
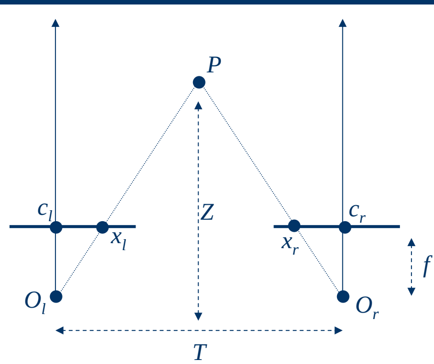
Basic Stereo Derivations



Derive expression for Z as a function of x_l, x_r, f and T

Basic Stereo Derivations



$$x_l = f\frac{X}{Z} \qquad x_r = f\frac{X}{Z}$$

$$\frac{T + x_l - x_r}{Z - f} = \frac{T}{Z}$$

$$Z(T + x_l - x_r) = T(Z - f)$$

$$ZT + Zx_l - Zx_r = TZ - Tf$$

$$\boxed{Z = f\frac{T}{x_r - x_l}}$$

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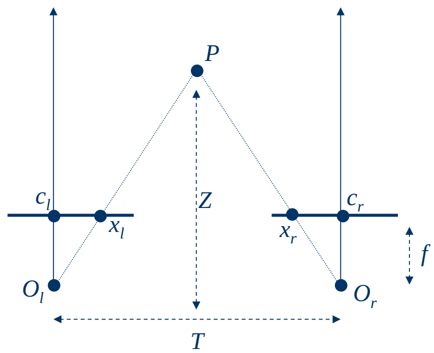
Basic Stereo Derivations



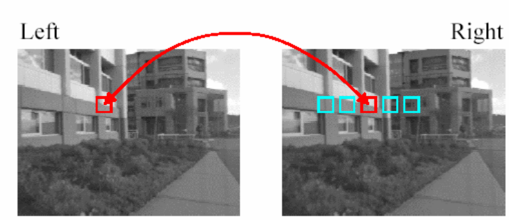
$$Z = f\frac{T}{x_r - x_l}$$

$$d = x_1 - x_2 \quad \text{disparity}$$

$$Z = f\frac{T}{d}$$

Token Based Stereo

- Detect token
  - Corners, interest point, edges
- Find correspondences
- Interpolate surface



Finding Correspondences

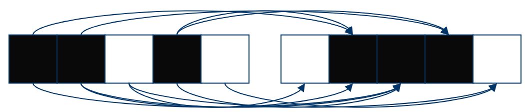
- Due to epilolar constraint stereo is a 1D search problem
  - Marr-Poggio
  - Correlation based stereo methods
  - Barnard's algorithm

Marr-Poggio Algorithm

- One of first stereo methods
- Applicable to image composed of pixels with randomly selected black or white colors
- Constraints:
  - **Compatibility:** white should match with a white
  - **Continuity:** neighbors should have similar disparity
  - **Uniqueness:** a pixel in left image matches with only one pixel on right image

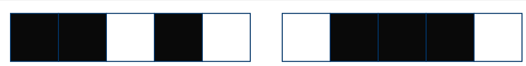
Marr-Poggio Algorithm

- Correspondence map
- Search is along the rows
- 1D case



	1	2	3	4	5
d=2	1	1	1		
d=1	1	1			
d=0		1		1	1
d=-1				1	
d=-2			1	1	

Initial correspondence array



Marr-Poggio Algorithm

- Iteratively update correspondence array using

1D Case

$$C^n(x,d)=\begin{cases}1 & \left\{\sum_{a=-w,a\neq 0}^{w} C^{n-1}(x+a,d) - \lambda \sum_{i=-D,i\neq 0}^{D} C^{n-1}(x\pm i, d+i) - C^0(x,d)\right\} > T \\ 0 & otherwise\end{cases}$$

excitatory term

max. allowed disparity

inhibitory term for continuity

2D Case

$$C^n(x,d)=\begin{cases}1 & \left\{\sum\sum_{|x-p|+|y-q|\leq w} C^{n-1}(x+p,y+q,d) - \lambda \sum_{i=-D,i\neq 0}^{D} C^{n-1}(x\pm i, y, d+i) - C^0(x,y,d)\right\} > T \\ 0 & otherwise\end{cases}$$

Correlation Based Stereo Methods

- Depth is computed only at tokens and interpolated/extrapolated to remaining pixel
- Disparity map is constructed based on a correlation measure

SSD $= \sum\sum(I_{t+1} - I_t)^2$

AD $= \sum\sum|(I_{t+1} - I_t)|$

CC $= \sum\sum I_{t+1} I_t$

NC $= \frac{\sum\sum(I_{t+1}.I_t)}{\sqrt{\sum\sum I_t.I_t}}$

MC $= \frac{1}{64\sigma_{t+1}\sigma_t}\sum\sum(I_{t+1} - \mu_{t+1})(I_t - \mu_t)$

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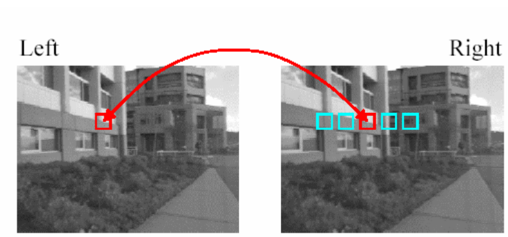
Correlation Based Stereo Methods

- Once disparity is available compute depth using $Z = f\frac{T}{d}$



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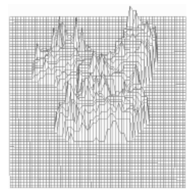
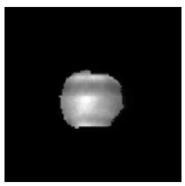
Examples

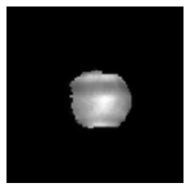
Left Image

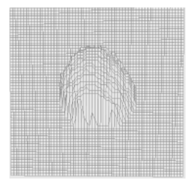
Right Image



Depth Map







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Barnard's Stereo Method

- Similar intensity
  - Similar to brightness constraint
- Smoothness of disparity

$$\mathrm{E} = \sum_{i=-1}^{1}\sum_{j=-1}^{1}\left\| I_{left}(x+i, y+j) - I_{right}(x+i+D(x,y), y+j)\right\| + \lambda \left\| \nabla D(x,y) \right\|$$

$$\nabla D(x,y) = \sum_{i=-1}^{1}\sum_{j=-1}^{1}\left| D(x+i, y+j) - D(x,y) \right|$$

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Barnard's Stereo Method

- Energy can be minimized using brute force search
  - Let max allowed disparity is 10 pixels
  - For every point we have to search a *circle* of radius 10
    - Aprox. 80 possible associations per pixel
    - For 128x128 image we will have $2^{(7+7+6)}=2^{20}$ associations
- We can select any minimization technique
  - Barnard choose simulated annealing

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Simulated Annealing

- Select a random state S (disparities)
- Select a high temperature
  - Select random S'
  - Compute $\Delta E=E(S')-E(S)$
  - If $(\Delta E<0)$ $S \leftarrow S'$
  - Else
    - $P \leftarrow \exp(-\Delta E/T)$
    - $X \leftarrow$ random(0,1)
      - If $X<P$ then $S \leftarrow S'$
    - If no decrease in several iterations lower T

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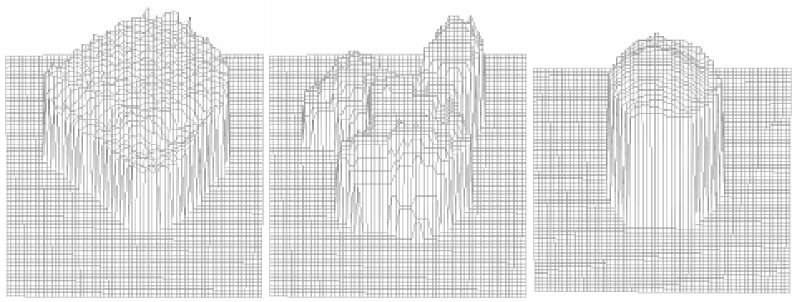
Examples



bread toy apple

Reading

- Trucco & Verri
  - Section 7.1 (pages 140-150)
- Lecture notes
  - Chapter 6

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