The arithmetic coders used in JPEG, JPEG 2000 and JBIG are called QM-coder. It handles only binary strings or input and it is designed for simplicity and speed. It uses approximation for multiplication operation, fixed-precision integer arithmetic with renormalization of the probability interval from time to time.

The main idea of the QM-coder is to classify the input bit as More Probable Symbol (MPS) and Less Probable Symbol (LPS). Before the next bit is input, the QM-coder uses a statistical model (using a context, typically a two-dimensional context of black and white pixel in an image) to predict which one of the bits (0 or 1) will be the MPS. If the predicted MPS bit does not match with the actual bit, then the QM-coder will classify this as LPS; otherwise, it will continue to be classified as MPS. The output of the coder is simply a stream of MPS or LPS, which are assigned probability values dynamically to be used by the simplified arithmetic coder. The decoder has only the knowledge of whether the next predicted bit is MPS or LPS. It uses the same statistical model as that of the encoder to obtain the actual values of the bit. Recall the range update equations we used for arithmetic coding.

\[
\begin{align*}
    \text{NewLow} &= \text{Low} + (\text{High} - \text{Low}) \times \text{CumLowBound}(a_i) \\
    \text{NewHigh} &= \text{Low} + (\text{High} - \text{Low}) \times \text{CumHighBound}(a_i) \\
\end{align*}
\]

Therefore,
\[
\begin{align*}
    \text{NewHigh} - \text{NewLow} &= (\text{High} - \text{Low}) \\
    [\text{CumHighBound}(a_i) - \text{CumLowBound}(a_i)] &= (\text{High} - \text{Low}) p(a_i)
\end{align*}
\]
If we use a variable $A$ to denote the range, we have the update equation for each interaction as

$$A_\text{new} = Ap(a_i)$$

Let us assign a probability $Q_e$ to LPS and assign the lower interval to MPS with

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Probability</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>LPS</td>
<td>$Q_e$</td>
<td>$[1-Q_e,1]$</td>
</tr>
<tr>
<td>MPS</td>
<td>$1-Q_e$</td>
<td>$[0,1-Q_e]$</td>
</tr>
</tbody>
</table>

Thus, the occurrence of an $MPS$ symbol will result in the update equation.

$$\text{Low}: = \text{Low}$$
$$A: = A (1 - Q_e)$$

And the occurrence of an $LPS$ symbol will result in the update equation.

$$\text{Low}: = \text{Low} + A (1 - Q_e)$$
$$A: = AQ_e$$

The JBIG committee recommended that the multiplication could be avoided by approximating the value of $A$ to be close to 1. Then the update equations become

**For MPS**
$$\text{Low}: = \text{Low}$$
$$A: = 1 - Q_e$$

**For LPS**
$$\text{Low}: = \text{Low} + 1 - Q_e$$
$$A: = Q_e$$

In order not to violate the assumption that $A$ is close to 1, whenever $A$ goes below 0.75 the QM coder goes through a series of rescaling until the value of $A$ goes higher than 0.75. The rescaling operation is simply doubling which corresponds to a left shift if $A$ is represented in
binary. The same rescaling must be applied to low. *The bits shifted out of the buffer containing the value of A constitute the encoder output.* Rescaling happens every time *LPS* occurs. For *A*, the rescaling occurs only if its value dips below 0.75. For further details of rescaling, integer implementation, decoding details and worked out examples, read pp. 118-122 (Salomon).

**JBIG**

(Only a summary of major points will be mentioned here; Read Sections 4.6.1 and 4.6.2 from Sayood and Section 4.9 from Salomon for details.)

- Uses “progressive” transmission of images which exist in different resolutions in “layers”, but avoids duplication of data by a technique called “deterministic prediction”.

- Although JBIG is designed for bi-level images, it can handle any “grayscale” image by separating the bit planes and compressing each plane separately.

- Uses “multiple arithmetic” coders (QM-coders). This is possible because the arithmetic coders can separate the modeling and the coding parts. There are 1024 or 4096 such models (corresponding to the number of possible pixel values that define the context) working in parallel. Each model is nothing but an entry of a probability value of a black pixel for the given pixel. This also reduces the source entropy because an effective 10th or 12th order Markov model is used.

- The lowest resolution layer uses either a 2-line or a 3-line context. Higher resolution layers context templates assume the availability of lower resolution layers.
The circled X’s represent lower resolution values already known to the next higher layer. The pixel marked “A” stands for adaptive pixel (AP) whose position can be specified relative to “?” left or above it by two bytes giving the values of offsets in positioning “A”. A cannot occupy any position occupied by an “X”. This feature is used by sophisticated JBIG encoder which may handle, for example, half-tone images where typically a black pixel normally has another black pixel three rows above it.