Restrictions: Help in Documenting Client Code Under a Verified Software Paradigm

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ABSTRACT

A novel programming language construct, restrictions, provides a mechanism to document abstract invariants of program variables and also may simplify program correctness proofs of the use of components. Examples illustrating the use and utility of restrictions are presented.

1. INTRODUCTION

It has long been claimed in some circles that software professionals cannot be expected to write mathematically rigorous descriptions of their code such as formal specifications and loop invariants [1]. This contention arguably underestimates the capabilities of software professionals—after all, most of them have not been *taught* either why or how to write such annotations, so it is not surprising they are currently unequipped to do so. Nonetheless, the perception has led to exploration of some promising mitigating techniques that might be useful under a verified software paradigm. One approach involves inferring invariants (*e.g.,* loop invariants) either by dynamic or static analysis of code [2, 3, 4, 5, 6]. A complementary approach involves minimizing what needs to be written in mathematical language by providing special syntax for certain situations: syntax that looks more familiar and code-like to software developers. For instance, rather than demanding that the post-condition of an operation include a clause like `x' = x`, `x = old(x)` or `x = #x` to specify that the value of `x` does not change, most specification languages have tailor-made syntax for documenting this. JML [7] uses a `modifies` clause to list operation parameters whose values the parameters point to might be changed during the operation body. RESOLVE [8] offers (among others) a `restores` parameter "mode" to state that an operation parameter, while it might change temporarily during the body of the operation, has the same value at the end of the operation body as it had at the beginning.

Such mechanisms incrementally reduce the mathematical annotation burden for the software professional. It is not yet clear how effective the invariant-inference approach will be under a verified software paradigm for component-based software; when automated verification does not succeed, it will be critical for a human to understand these invariants in order to repair the code, the annotation or both. This means that inferred invariants should be not only technically correct but also comprehensible to the software professional, who will ultimately be responsible for at least reading and likely for modifying formal mathematical descriptions of software behavior. Some human input into writing invariants and other assertions therefore seems unavoidable.

This paper describes a modest advance down the special-syntax path: providing language constructs to reduce the annotation burden. It focuses on relationships between abstract invariant properties of individual variables that hold during an entire code segment and loop invariants within that segment. We observe that two kinds of properties must be included in a loop invariant to verify software. The first kind arise from the desire to treat a loop as a single statement in straight-line code for verification and reasoning purposes. These document the behavior of the loop by stating what it does not change; they are intimately tied to the loop and are local to it. The second kind arise from the need to maintain continuity of abstract invariants on variable values. These properties are often incidental to a particular loop yet are critical pieces of the loop invariant. For example, when using memoization to avoid re-computation of a function with a Java `Map`, one abstract invariant on the `Map`'s value is that if a key is defined then the value associated with that key is the function applied to the key. This information must be in the loop invariant for any loop involving the `Map`, because this property is true before the loop is encountered, is maintained by the loop, and might be intended to persist after the loop has terminated. This restricted set of `Map` values is known *a priori* by the software developer independently of any loops, and it can and should be documented. If the documentation is formal, its connection to the code can be verified. In other words, this documentation not only records the software developer's reasoning but—in a verified software paradigm—also can be used to check that the reasoning is correct.

The contribution of this paper is a programming language construct, restrictions, that can be used to document *abstract invariant properties of individual variables over segments of imperative code* without introducing new programmatic types. This construct allows for reuse of these invariants. Moreover, it implicitly provides guidance to the verifier by "factoring" potentially complicated verification conditions (VCs) into conceptually simpler VCs. The overabundance of assumptions in VCs has been reported [9, 10] as a problem for back-end provers.

Restrictions are presented in the context of the RESOLVE programming language, a research language designed for verifiability. Specifically, RESOLVE has clean semantics, and provides syntactic slots for contracts and mathematical annotations of various kinds. However, the restriction construct should be adaptable to other programming languages with little change, so long as the specification language language can express and ensure frame properties, such as JML [7] or Dafny [11].

The paper is structured as follows. Section 2 presents a simple motivating example (in C++ rather than RESOLVE). Section 3 includes a summary of the features and syntax of RESOLVE needed to explain restrictions. An introduction to restrictions in Section 4 features an in-depth example using sorting. Related work is discussed in Section 5, with conclusions in Section 6.

2. MOTIVATING EXAMPLE

Consider code that computes x^p where x is a double and p is a positive integer; see Figure 1(a). It computes x^p by first computing x^{2^k} where k is the largest natural number that satisfies $2^k \leq p$ and then making a recursive call to finish the job. In this particular implementation, `q` always equals $2^{k'}$ where k' is some non-negative integer, and this property holds both as a loop invariant and, more generally, as an invariant on `q` throughout the code. We argue that this invariant can and should be documented.

One method a software professional can use to document the invariant on `q` is to add extra assertions in the code. At every line where the invariant holds, she `assert`s the invariant. Frame properties allow one to limit the number of such statements needed, by using them only after a modification to a variable under consideration. This documents the invariant on `q`, but it is rather clumsy and the annotation burden is high. Restrictions (Section 4) are a construct to document the claims for this code more clearly and to reduce the annotation burden. Figure 1(b) shows what the code might look like in this situation. The loop invariant is simplified and the invariant on `q` is explicit.

3. RESOLVE OVERVIEW

As mentioned in the introduction, RESOLVE is an imperative and component-based research language designed for verifiability, performance and understandability [8]. The language has reference semantics with the following critical qualification that is enforced as a consequence of the language primitives: no aliasing of references across component boundaries is possible, *i.e.,* there is no inter-component aliasing. Practically speaking, aliasing even within a single component implementation is rarely used except in implementations of a few low-level library components. This language restriction, a frame property, provides the illusion or effect of value semantics for any client usage (*i.e.,* code written by users) of any component. Each component has a mathematical model of its behavior described in a contract, as illustrated in Figure 2 by a `Queue` contract. The mathematical model is explicit in the type declaration. Each operation has a formal description of behavior in terms of the mathematical model via standard `requires` and `ensures` clauses. The `control` return type is used within `if`/`while` conditions. One more restriction simplifies verifiability: all program function operations must behave as mathematical functions and must restore their arguments. This is superficially similar to the restriction that functions be "pure" in JML [7] or Dafny [11]. The difference is that RESOLVE program functions still may not be used in specifications because RESOLVE rigorously separates and distinguishes mathematical entities (including definitions of mathematical functions) from programming entities (including program operations, *i.e.,* program functions, that happen to have functional behavior).

```
double Power (double x, int p)
{
    double result = x;
    int q = 1;
    while(q <= p/2)
    /*!
        updates result, q
        maintains
            result = x ^ q and
            q <= p and
            there exists k : integer
                (q = 2 ^ k)
        decreases
            p - q
    !*/
    {
        q *= 2;
        result *= result;
    }
    if (p - q > 0)
    {
        result *= Power(x, p-q);
    }
    return result;
}
```

(a) Original version

```
double Power (double x, int p)
{
    double result = x;
    int q = 1;
    /*! restrict q to be a power of 2 !*/
    while(q <= p/2)
    /*!
        updates result, q
        maintains
            result = x ^ q and
            q <= p
        decreases
            p - q
    !*/
    {
        q *= 2;
        result *= result;
    }
    if (p - q > 0)
    {
        result *= Power(x, p-q);
    }
    return result;
}
```

(b) Documented with a restriction

Figure 1: Code to compute x^p

Behavioral extensions to abstract components, such as the `Concatenate` extension in Figure 2, are specified via con-

```
contract QueueTemplate (type Item)

    uses UnboundedIntegerFacility

    math subtype QUEUE_MODEL is string of Item

    type Queue is modeled by QUEUE_MODEL
        exemplar q
        initialization ensures
            q = empty_string

    procedure Enqueue (updates q: Queue,
                       clears x: Item)
        ensures
            q = #q * <#x>

    procedure Dequeue (updates q: Queue,
                       replaces x: Item)
        requires
            q /= empty_string
        ensures
            #q = <x> * q

    function IsEmpty
            (restores q: Queue): control
        ensures
            IsEmpty = (q = empty_string)

end QueueTemplate

contract Concatenate enhances QueueTemplate

    procedure Concatenate (updates p: Queue,
                           clears q: Queue)
        ensures
            p = #p * #q

end Concatenate
```

Figure 2: `QueueTemplate` contract and `Concatenate` extension

tracts and ordinarily implemented by layering on other components' contracts. Another extension of a `Queue` type is the operation `Sort`. Sorting has been studied by the computer science community since the field's inception; in the past few years there has been significant work on inferring loop invariants [2, 3, 4, 5, 6] among other work on verification of sorting algorithms. For the purposes of demonstrating the utility of restrictions, sorting therefore serves as an appropriate standard benchmark that naturally involves variables with abstract invariants beyond any of the generic abstract data type (ADT) invariants of its variables.

3.1 Sort Specification

Since the `QueueTemplate` component is generic, *i.e.,* parameterized by a type `Item`, the contract of a `Sort` operation should also be generic. Figure 3 shows the requisite restriction on the ordering relation `ARE_IN_ORDER` to be used in sorting, namely that `ARE_IN_ORDER` is a total pre-order.

Figure 3 shows the mathematical definitions used to specify sorting, given `ARE_IN_ORDER`. `OCCURS_COUNT` is a mathematical function that returns the number of times a given `Item` appears in a string; it is used to construct the other mathematical definitions. This allows the contract to be specific about the value of the outgoing `Queue`: not only are the values of items in the outgoing `Queue` the same as in the incoming queue, but the number of times each appears is the same. `IS_PRECEDING` is a binary predicate and holds on two strings if and only if every item in the first string is related by `ARE_IN_ORDER` to every item in the second string; intuitively, every item in the first string is "no larger" than every item in the second. `IS_NON_DECREASING` is a unary predicate that is true if and only if every consecutive pair of `Items` in the string are related by `ARE_IN_ORDER`. Finally, `IS_PERMUTATION` is a binary predicate on strings that is true if and only if the number of occurrences of every `Item` is the same in the first string and in the second.

```
contract Sort (
  definition ARE_IN_ORDER (x: Item,
                           y: Item): boolean
       satisfies
        for all z: Item
            ((ARE_IN_ORDER (x, y)  or
              ARE_IN_ORDER (y, x))  and
             (if (ARE_IN_ORDER (x, y)  and
                  ARE_IN_ORDER (y, z))
              then ARE_IN_ORDER (x, z))))
    enhances QueueTemplate


    definition OCCURS_COUNT (
        s: string of Item,
        i: Item
    ) : integer
       satisfies
           if s = empty_string
           then OCCURS_COUNT (s, i) = 0
           else
               there exists x: Item,
                            r: string of Item
                 ((s = <x> * r) and
               (if x = i
               then OCCURS_COUNT (s, i) =
                    OCCURS_COUNT (r, i) + 1
               else OCCURS_COUNT (s, i) =
                    OCCURS_COUNT (r, i)))

    definition IS_PRECEDING (
         s1: string of Item,
         s2: string of Item
    ) : boolean
        is for all i, j: Item
             where (OCCURS_COUNT (s1, i) > 0 and
                    OCCURS_COUNT (s2, j) > 0)
             (ARE_IN_ORDER (i, j))

    definition IS_NON_DECREASING (
         s: string of Item
    ) : boolean
        is for all a, b: string of Item
             where (s = a * b)
             (IS_PRECEDING (a, b))

    definition IS_PERMUTATION (
        s1: string of Item,
        s2: string of Item
    ) : boolean
      is for all i: Item
           (OCCURS_COUNT (s1, i) =
            OCCURS_COUNT (s2, i))


    procedure Sort (updates q: Queue)
        ensures
            IS_PERMUTATION (q, #q)  and
            IS_NON_DECREASING(q)

end Sort
```

Figure 3: `Sort` extension to `QueueTemplate`

The contract specification of a `Sort` operation is given in Figure 3. The `Sort` operation takes a `Queue` and returns with the property that the outgoing string `q` is a permutation of the incoming string and the outgoing string is non-decreasing with respect to `ARE_IN_ORDER`.

3.2 Quicksort Implementation

We present an implementation of the `Sort` operation using quicksort in Figure 4. Our implementation partitions a non-empty incoming queue into two queues (`q` and `qBig`) and a partitioning element (`partitionElement`) with the property that every `Item` in `q` is in order with `partitionElement` and `partitionElement` is in order with every item in `qBig`. Each of the smaller queues is sorted recursively and `q`, `partitionElement`, and `qBig` are all concatenated to obtain the final, sorted queue. Besides the loop invariant and other mathematical annotations, this code is similar to code in most other languages. The loop invariant documents the insight of the algorithm, namely the ordering relationships among the variables `partitionElement`, `q` and `qBig`, as expressed formally via `IS_PRECEDING` and `IS_PERMUTATION`. The programmer's justification for termination is given by the `decreases` clause (*i.e.,* progress metric).

A local operation `Partition` is used to split a queue according to the quicksort algorithm. The `:=:` operator is the "swap " operator [12, 13]. This exchanges the values of its two arguments, and is a key aspect of avoiding aliasing while preserving efficiency.

4. INTRODUCTION TO THE SYNTAX AND SEMANTICS OF RESTRICTIONS: SORTING EXAMPLE

First we examine the issues involved in defining of restrictions (the new programming language construct), and then the issues in the usage of restrictions in client code. Code presented in this section is analogous to the code in section 3.1 except it uses restrictions.

4.1 Restrictions

We create three restrictions for this example, one for each different abstract invariant maintained by specific uses of `Queue`s in quicksort. The first invariant is that a `Queue` is sorted, *i.e.,* it is an `OrderedQueue`. The other invariants relate the `Item`s in a `Queue` to another `Item`. These invariants arise during the `Partition` implementation and simply relate `qSmall` to `p` and `qBig` to `p` by `ARE_IN_ORDER`; more specifically every `Item` in `qSmall` is in order with `p` and `p` is in order with every `Item` in `qBig`. For each operation that is called on any `Queue` that satisfies one of these properties, the programmer reasons that the abstract invariant is not broken by the operation call. The proof boils down to the question: if the operation is executed, does the new value of the variable still satisfy the restriction? With this intuition in mind, we show the contracts for the restrictions corresponding to these ideas in Figure 5.

A restriction is declared relative to one or more existing contracts, *e.g.,* `QueueTemplate`. Operations of the underlying contract may be given additional `requires` and `ensures`. The restriction is given by a predicate where parameters are of the specified types. Since functions may not "break" the invariant—they cannot change the abstract value of any argument—they are always available to be used with a program type in any restriction.

```
realization QuickSort (
   function AreInOrder (restores i: Item,
                        restores j: Item): control
         ensures
             AreInOrder = ARE_IN_ORDER (i, j)
   ) implements Sort for QueueTemplate

   uses Concatenate for QueueTemplate

   local procedure Partition (updates qSmall: Queue,
                              replaces qBig: Queue,
                              restores p: Item)
       ensures
           IS_PERMUTATION (qSmall * qBig, #qSmall)
           and IS_PRECEDING(<qSmall, <p>)
           and IS_PRECEDING(<p>, qBig)

       variable tmp: Queue
       Clear (qBig)
       loop
           updates qSmall, qBig, tmp
           maintains
               IS_PERMUTATION (qSmall * qBig * tmp,
                               #qSmall * #qBig * #tmp)
               and IS_PRECEDING(tmp, <p>)
               and IS_PRECEDING(<p>, qBig)
           decreases |qSmall|
       while not IsEmpty (qSmall) do
           variable x: Item
           Dequeue (qSmall, x)
           if AreInOrder (x, p) then
               Enqueue (tmp, x)
           else
               Enqueue (qBig, x)
           end if
        end loop
        qSmall :=: tmp
   end Partition

   procedure Sort (updates q: Queue)
       decreases |q|

       if not IsEmpty (q) then
           variable partitionElement: Item
           variable qBig: Queue

           Dequeue (q, partitionElement)
           Partition (q, qBig, partitionElement)
           Sort(q)
           Sort(qBig)

           Enqueue (q, partitionElement)
           Concatenate (q, qBig)
       end if
   end Sort
end QuickSort
```

Figure 4: `Quicksort` implementation of `Sort` extension to `QueueTemplate`

```
contract OrderedQueueTemplate
   restricts QueueTemplate

   restriction OrderedQueue(q: Queue)
       is (IS_NON_DECREASING(q))

   procedure Enqueue(q: Queue, x: Item)
            under restriction
               OrderedQueue(q)
            also requires
               IS_PRECEDING (q,<x>)

   procedure Dequeue(q: Queue, x: Item)
            under restriction
               OrderedQueue(q)
            also ensures
               IS_PRECEDING (<x>, q)
end OrderedQueueTemplate

contract SmallValueQueueTemplate
   restricts QueueTemplate

   restriction SmallValueQueue(q: Queue,
                                max : Item)
       is (IS_PRECEDING(q, <max>))

   procedure Enqueue(updates q : Queue,
                     clears x : Item)
            under restriction
               SmallValueQueue(q, max)
            also requires
               ARE_IN_ORDER(x, max)

   procedure Dequeue(updates q : Queue,
                     replaces x : Item)
            under restriction
               SmallValueQueue(q, max)
            also ensures
               ARE_IN_ORDER(x, max)
end SmallValueQueueTemplate

contract LargeValueQueueTemplate
   restricts QueueTemplate

   restriction LargeValueQueue(q: Queue,
                                min : Item)
      is (IS_PRECEDING( <min>, q))

   procedure Enqueue(updates q : Queue,
                     clears x : Item)
             under restriction
                LargeValueQueue(q, min)
             also requires
                ARE_IN_ORDER(min, x)

   procedure Dequeue(updates q : Queue,
                     replaces x : Item)
             under restriction
                LargeValueQueue(q, min)
             also ensures
                ARE_IN_ORDER(min, x)
end SmallValueQueueTemplate
```

Figure 5: `OrderedQueue`, `SmallValueQueue` and `LargeValueQueue` restrictions

Conceptually, the `also requires` clauses are conjoined with the original `requires` clauses for the operation. These are used by the programmer to ensure both that the restriction is maintained by the operation, and to document conditions under which it is safe to call the operation while still maintaining the invariant. The `also ensures` clauses strengthen the previous postconditions. In the `OrderedQueue` contract, `Dequeue`'s `also ensures` clause gives information about how the dequeued item relates to items that remain in the `OrderedQueue`.

Since programmers may need some help in making sure that their reasoning process is correct, the compiler should generate VCs corresponding to the correctness of the restriction contract. The contract's correctness condition is that if an operation is invoked in a state satisfying the variable restrictions and the `requires` clause, and the operation completes successfully, then the restriction is still satisfied by the updated variables; any `also ensures` clauses must also be satisfied. More concretely, each operation's invocation can be assumed to occur in a state in which the restriction, the original `requires` clause, and the `also requires` clause hold. By a process similar to datatype induction, these VCs are generated just once for the contract. (Notice that this construction leaves the initialization of restrictions to a client-side activity and is discussed in Section 4.2.) The general form of the generated VCs, where s is a variable of the mathematical model of the restriction, $args$ is the list of arguments to the operation, and $'$ indicates a fresh variable, is given by:

$$\begin{gathered} \mathit{restriction}(s') \wedge \mathit{requires}_{\mathit{original}}(s', \mathit{args}') \wedge \\ \mathit{requires}_{\mathit{also}}(s', \mathit{args}') \wedge \\ \mathit{ensures}_{\mathit{original}}(s', \mathit{args}', s, \mathit{args}) \\ \implies \mathit{restriction}(s) \wedge \mathit{ensures}_{\mathit{also}}(s', \mathit{args}', s, \mathit{args}) \end{gathered}$$

4.2 Client Usage of Restrictions

The updated `Sort` contract, shown in Figure 6, is almost the same as the original contract. The difference is that the `Queue` formal parameter `q` is restricted to satisfy the restriction `OrderedQueue` when the operation returns. The formal parameter `q` must be of type `Queue`; when `Sort` returns, `q` conforms to the restriction `OrderedQueue` (checked as a proof obligation). We can omit the `IS_NON_DECREASING(q)` from the `ensures` clause, since it is subsumed by the restriction.

```
contract Sort (
...

   procedure Sort (updates q: Queue)
       establishes restriction
           OrderedQueue(q)
       ensures
           IS_PERMUTATION (q, #q)

end Sort
```

Figure 6: `Sort` extension to `QueueTemplate` using restrictions

This reduces the mathematical annotation burden on the programmer. The `restriction` annotation in the formal parameters need not be checked by the static type system. It is equivalent to having the type restriction in the `ensures` clause for that variable. We examine this issue in more depth in the discussion of the `Sort` operation.

Figure 7 shows an additional operation `Concatenate` defined on `Queues` that is used by `OrderedQueues`. The `also requires restriction` slot is used in this contract to indicate that two variables, `q1` and `q2` satisfy the `OrderedQueue` restriction.

```
contract OrderedQueueConcatenate
 restricts Concatenate for QueueTemplate

   procedure Concatenate(updates q1: Queue,
                         clears q2: Queue)
          under restriction
              OrderedQueue(q1) and
              OrderedQueue(q2)
          also requires
              IS_PRECEDING(q1, q2)
end OrderedQueueConcatenate
```

Figure 7: `OrderedQueueConcatenate` Restriction

The `Partition` operation uses the `SmallValueQueue` and `LargeValueQueue` restrictions. The code for performing the partition operation is given in Figure 8. In the contract of `Partition`, the `ensures` clause and loop invariant are simplified by the use of the `establishes restriction` annotation. Otherwise, the code is similar to the original version in Section 3.1.

Recall that in the contracts of restrictions, there were no VCs generated for initialization; that piece is left to the clients or users of the restriction. So, when a variable of a particular type, say `Queue`, has a new restriction, say a `LargeValueQueue`, a VC is generated to make sure that that variable satisfies that restriction. For example, `confirm restriction LargeValueQueue(qBig, p)` generates a VC whose goal is `IS_PRECEDING(<p>, qBig)` and whose assumptions are those facts known at that point in the code, *e.g.,* resulting from path conditions, loop invariants, and contracts of other operations called. We note that not only is the specification of `Partition` simpler, but the loop invariant has been significantly simplified as well.

Figure 8 also shows the `Sort` implementation using restrictions and the modified `Partition` local operation. Except for the `confirm restriction` annotation, the code is exactly the same as the original version. The loop invariant is simplified as two conjuncts may be removed as the restrictions implicitly ensures the loop invariant.

Finally, to finish an earlier discussion about the implementation of the `expects restriction` or `establishes restriction` annotation in an operation parameter, one can implement the annotation by automatically translating it into a `requires` or `ensures` clause, respectively, in the operation contract. On every client use of the operation, the verification system adds a `confirm restriction` annotation after the call to reassert the restriction, generating one additional (simple) VC. This process can be invisible to the user, but simplifies the information needed for restrictions by avoiding carrying it across operation boundaries.

4.3 Evaluation

Appropriate use of restrictions may also help simplify proofs of VCs by making the VCs easier to prove. We examine the impact of restrictions on the difficulty of VCs as defined by [14]. In that work, VCs are categorized according to the number of hypotheses (H_0, H_1,...) and whether only logical rules (L), theory-specific knowledge (M) or local mathematical definitions (D) are needed to prove a VC. VCs that use fewer assumptions or require less mathematical knowledge are considered less difficult. The metrics are summarized in Figure 9.

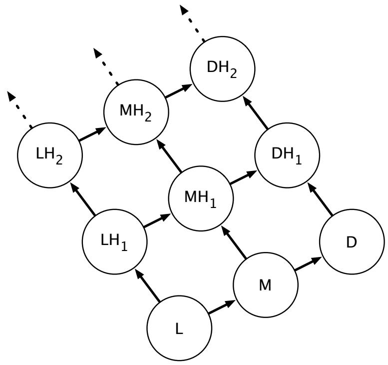
```
realization QuickSort (
....
uses OrderedQueueConcatenate for QueueTemplate

    local procedure Partition
                  (updates qSmall: Queue,
                   replaces qBig: Queue,
                   restores p: Item)
        ensures
            IS_PERMUTATION (qSmall * qBig,
                            #qSmall)
        establishes restriction
            LargeValueQueue(qBig,p) and
            SmallValueQueue(qSmall,p)

        variable tmp: Queue
        confirm restriction SmallValueQueue(tmp, p)
        Clear (qBig)
        confirm restriction LargeValueQueue(qBig, p)
        loop
            updates qSmall, qBig, tmp
            maintains
                IS_PERMUTATION (qSmall * qBig * tmp,
                                #qSmall * #qBig * #tmp)
            decreases |qSmall|
        while not IsEmpty (qSmall) do
            variable x: Item
            Dequeue (qSmall, x)
            if AreInOrder (x, p) then
                Enqueue (tmp, x)
            else
                Enqueue (qBig, x)
            end if
         end loop
         confirm restriction SmallValueQueue(qSmall, p)
         qSmall :=: tmp
    end Partition

    procedure Sort (updates q: Queue)
        decreases |q|

        variable qtmp: Queue
        qtmp :=: q
        confirm restriction OrderedQueue(q)
        if not IsEmpty (qtmp) then
            variable partitionElement: Item
            variable qBig: Queue

            Dequeue (qtmp, partitionElement)
            Partition (qtmp, qBig,
                          partitionElement)
            Sort(qtmp)
            Sort(qBig)

            Enqueue (qtmp, partitionElement)
            Concatenate (qtmp, qBig)
            q :=: qtmp
        end if
    end Sort
end QuickSort
```

Figure 8: Quicksort implementation of `Sort` using restrictions

The code presented in Section 3 without restrictions was compared to the code in this section with restrictions. The original quicksort implementation's most difficult VC was categorized as MH_6, while the restrictions version has VCs of difficulty at most MH_3. The MH_6 VC is particularly difficult; it arises from proving the second conjunct in the

Label	What is needed in the proof
L	Rules of mathematical logic
H_n	At most n hypotheses from the VC ($n > 0$)
M	Knowledge of mathematical theories used in the specifications
D	Knowledge of programmer-supplied definitions based on mathematical theories above

(a) VC classification



(b) Lattice of the VC classification

Figure 9: VC classification and diagram of category relationships (adapted from [14])

`ensures` clause of `Sort`:

$$\begin{array}{lcl}
1: & & is\_initial(partitionElement_2) \\
2: & \wedge & IS\_PERMUTATION(q_5 * qBig_5, q_4) \\
3: & \wedge & IS\_PRECEDING(q_5, \langle partitionElement_4 \rangle) \\
4: & \wedge & IS\_PRECEDING(\langle partitionElement_4 \rangle, qBig_5) \\
5: & \wedge & IS\_PERMUTATION(q_6, q_5) \\
6: & \wedge & IS\_NON\_DECREASING(q_6) \\
7: & \wedge & IS\_PERMUTATION(qBig_7, qBig_5) \\
8: & \wedge & IS\_NON\_DECREASING(qBig_7) \\
9: & \wedge & is\_initial(partitionElement_8) \\
10: & \wedge & \langle partitionElement_4 \rangle * q_4 \neq \Lambda \\
\hline
 & \rightarrow & IS\_NON\_DECREASING(q_6 * \\
 & & \langle partitionElement_4 \rangle * qBig_7)
\end{array}$$

The proof requires hypotheses 3 through 8, and is fairly involved; mathematical lemmas are needed, for instance, to conclude that hypotheses 3, 5 and 6 imply $IS\_PRECEDING(q_6, \langle partitionElement_4 \rangle)$. The corresponding VCs from restrictions are easier. The direct analog of the above VC, in particular, is in category LH_1, *i.e.,* the goal is one of the hypotheses. The proof of a VC arising from the call to `Concatenate` in the body of `Sort` is the most difficult: it is in the category MH_3.

Another VC in MH_3 arises from the `also requires` clause for `Concatenate`:

$$\begin{array}{lcl}
1: & & IS\_PRECEDING(q1_{original}, q2_{original}) \\
2: & \wedge & IS\_NON\_DECREASING(q1_{original}) \\
3: & \wedge & IS\_NON\_DECREASING(q2_{original}) \\
\hline
 & \rightarrow & IS\_NON\_DECREASING(q1_{original} * q2_{original})
\end{array}$$

The one-time, reusable proof of this VC is also in MH_3. However, it is a relatively easy proof to discharge; it is an algebraic lemma of string theory. This is the essence of the proof of the original VC. Proving these VCs with Isabelle [15] using a version of RESOLVE's string theory in an automatic mode [10] confirms that the MH_6 VC is hard to prove—Isabelle does not prove it automatically. The two MH_3 VCs are proved automatically. For this example, restrictions are able to simplify the code annotations and reduce the maximum difficulty of VCs generated from the resulting code.

We expect this empirical result to generalize; restrictions have the effect of adding "way-points" in the proofs of the VCs from code using restrictions. These way-points are created from input from the programmer; the `requires` and `ensures` clause are both modified to preserve the requisite invariant, thus ensuring that the way-point is useful for the justification of correctness. Moreover, the VCs generated from the declaration of a restriction should be syntactically simple with few assumptions and highly targeted—excellent candidates for proofs from general, reusable theorems.

We also expect that many restrictions will be reusable. For example, `OrderedQueue` may be used for any sorting algorithm implementation or client. Restrictions presented in this paper could be generalized to be usable in selection problems, *e.g.,* via a predicate parameter to `SmallValueQueue`. Even if we assume that restrictions turn out to not be reusable, there is still value in using them; restrictions document the reasoning behind why a particular block of code is correct, and, as such, aid readability by humans.

5. RELATED WORK

The idea of restrictions is similar to a core idea expressed in predicate subtypes, dependent types, refinement types and contract types [16, 17] (PDRCT). Depending on the exact setup of PDRCT used, proof obligations may be generated (such as Type Correctness Conditions (TCCs) in PVS) when converting from a type to a predicate subtype. In other setups the type checking system can infer many of the requisite properties. These ideas have been applied both to mathematical and programmatic domains. In any case, a restriction is different in that it entails modifying pre/post-conditions of operations to maintain the user-supplied invariant. Moreover, no new executable code need be emitted as a result of a restriction, which documents invariants and simplifies proofs of resulting VCs rather than defining a new type; a restriction is *not* a new type. However, the type inference and other algorithms used in contract and refinement types are largely absent; these could be added in the future using some of the existing work to alleviate some of the annotation burden on programmers.

The Jahob system [9] uses annotated Java source code as its source language. The annotation language has support for a proof language, with essentially full first-order prover functionality. There are first order proof commands, such as applying modus ponens, along with commands to perform local proofs. Invariants can be expressed as well. While Jahob's proof language is powerful, the proof commands are not natural for a software professional. Rather than learning a proof system, software professionals using restrictions think in terms of contracts and component invariants, concepts that are used in the normal course of programming.

Behavioral subtyping [18] uses a set of rules to ensure that a subtype can always be used in place of a supertype without violating a behavioral property of the client program. Contractually, the preconditions of any subtype operation may not be strengthened, postconditions may not be weakened, and invariants must be preserved. Restrictions impose different requirements; in particular, preconditions may be strengthened. The goal of restrictions is not to allow for substitution, but rather to indicate that during specific code segments (i.e., not necessarily for the entire lifetimes of variables) stronger abstract invariants hold for specific variables.

Object invariants [19] are defined over the *concrete* representation of the object; they denote consistency or other properties that relate specific fields or ownership of a particular field or object. Restrictions instead are over the *abstract* state of the objects, their cover story as represented in mathematics, rather than over any particular representation of the object's abstract state space. This feature ensures that restrictions can be used with any correct implementation of their underlying type, making them more reusable.

6. CONCLUSION

We have presented a programming language construct, restrictions, that helps address a limitation in current verification languages, namely the clumsiness of formally documenting client code, especially with loops. This construct, when applied to code similar to that shown in Section 4, provides a mechanism to separate out two uses of loop invariants, namely an abstraction of the behavior of a loop and a mechanism to maintain abstract invariants on variables. This approach not only can simplify VCs generated in client code, but also can result in reasoning reuse. This reuse happens both when restrictions are reused across clients, and even when there are multiple calls to a single restriction operation by a particular client.

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