Homework 2: Lambda Calculus

This homework is due Monday, Feb. 18 (in class).

Please do the problems by hand, and only use the lambda calculus interpreter to check your work. (Don't let the computer do your thinking for you.)

1. (10 points) Using the encoding given in the lambda calculus handout in the course reference pack for the natural numbers (i.e., the Church numerals), give lambda terms that implement:

   (a) multiplication, and

   (b) exponentiation.

2. (12 points) Do part (a) of exercise 13 (pages 57-58) of Schmidt's book, parts (i) through (vi). Note that you are only to use the β rule for this part.

3. (10 points) Do part (b) of exercise 13 (page 58) of Schmidt's book.

4. (extra credit) Do part (d) (i) of exercise 13 on page 58 of Schmidt's book.

5. (10 points) Some fun with combinators. Let

$$\begin{aligned} I &\stackrel{\text{def}}{=} \lambda x\,.\,x \\ K &\stackrel{\text{def}}{=} \lambda x\,.\,\lambda y\,.\,x \\ S &\stackrel{\text{def}}{=} \lambda x\,.\,\lambda y\,.\,\lambda z\,.\,xz(yz) \\ B &\stackrel{\text{def}}{=} S(KS)K. \end{aligned}$$

   Derive the normal forms (if any for the following):

   (a) $(SKK)x$

   (b) $Bxyz$

   (c) $(S(BBS)(KK))xyz$

6. (extra credit) Prove that application in the lambda calculus is not associative.

7. (extra credit) Show that $(\lambda y\,.\,(\lambda x\,.\,M))N = (\lambda x\,.\,(\lambda y\,.\,M)N)$

8. (extra credit) Prove that every term of the simply-typed lambda calculus has a normal form.

9. (extra credit) Suppose a symbol of the λ-calculus alphabet is always 0.5 cm wide. Write down a λ-calculus term with length less than 20 cm having a normal form with length at least $10^{10^{10}}$ lightyear. (Note: the term must have a normal form.) The speed of light is $c = 3 \times 10^{10}$ cm/sec.