Homework 4: Basics of Denotational Semantics

This homework is due Monday, March 18.

Feel free to do the denotational semantics questions with pencil and paper. Typesetting denotational semantics is hard. However, if you want to type up your homework solutions using LaTeX, you may want to use the files `~leavens/include/math-notations.tex` and `~leavens/include/denotational-semantics.tex`.

An interesting way to check your work is to use mini-fx to "implement" your solutions; type-checking alone is a valuable aid. If you do this, you can turn in mini-fx code as extra credit. However, beware that mini-fx is a strict language, whereas denotational semantics is done in a normal-order language (λ-calculus).

1. (5 points) Do exercise 12 on page 32 of Schmidt's book.
2. (13 points) Do exercise 3 on page 55 of Schmidt's book.
3. (6 points) Do exercise 4, parts (d), (e) and (f) on page 55 of Schmidt's book.
4. (10 points) Design a semantic algebra for the notion of "time" appropriate for a real-time system.
5. (extra credit only) Design a semantic algebra for the notion of "space" in a real-time system.
6. (extra credit only) Do exercise 12 (as many parts as you wish) on page 57.
7. (20 points) This problem requires you to add constructs `or` and `disjunction` to the denotational semantics of FL. The semantics of these constructs is as follows:
   - `(or` e_1 e_2`)` returns the value of e_2 if e_1 evaluates to `#f`; returns `#t` if e_1 evaluates to `#t`; and otherwise is undefined. (e_2 is not evaluated when e_1 is `#t`.)
   - `(disjunction` e_1 e_2`)` returns `#f` if both e_1 and e_2 evaluate to `#f`; returns `#t` if e_1 and e_2 evaluates to some other combination of boolean values; and is otherwise undefined.

   You are to extend the valuation functions in the denotational semantics of FL to handle these new constructs.
8. (32 points) Do exercise 8 on page 72 of Schmidt's book. (Hint: you may want to try exercises 4, 5, and 6 first, which you can hand in for extra credit if you wish.)