An Evolutionary Game Theoretic Framework for Coexistence in Cognitive Radio Networks

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Abstract—**Cognitive Radio Networks (CRN) employ coexistence protocols for spectrum sharing when collocated in a given region. Existing coexistence protocols do not take into consideration the fact that available spectrum bands vary significantly in their characteristics and quality they provide which makes some channels of the spectrum more attractive than others. In this paper, we analyze this situation from an evolutionary game theoretic perspective and show how CRNs would evolve their strategies of contending for disparate spectrum resources. We derive the equilibrium state for CRNs' spectrum sharing game and show that the population mix in equilibrium cannot be invaded by a mutant strategy which is greedier than the incumbent strategy and is therefore an evolutionarily stable strategy (ESS). We also derive the replicator dynamics of the proposed evolutionary game which represent how players learn from payoff outcomes of their strategic interactions and modify their strategies at every stage of the game. Since all players approach the ESS based solely on the common knowledge payoff observations, the evolutionary game can be implemented in a distributed manner. Simulation results show that the replicator dynamics enable strategic choices of CRNs to converge to ESS and also make them robust against changing network conditions.**

Index Terms—**Cognitive Radio, Coexistence, Game Theory.**

I. INTRODUCTION

Existence of a wide gap in the demand and supply of wireless spectrum resource forced regulatory bodies such as the FCC to allow un-licensed access to spectrum bands, also referred to as the TV white spaces, otherwise licensed to the Primary Users (PU) in an opportunistic and non-interfering basis [1]. This has given rise to a challenging as well as an exciting type of networks called the Cognitive Radio Networks [2], [3]. CRNs employ Dynamic Spectrum Access (DSA) to ensure that their use of spectrum does not cause interference to the PU and that all spectrum opportunities are utilized to the maximum. In many cases, a central entity such as a CRN base station controls the use of specific channels in a spectrum band within the CRN: IEEE 802.22 wireless regional area networks (WRANs) [2], [3] is an example of such a network. However, there may be many CRNs collocated in a region all of whom compete for access to the available channels, a situation called self co-existence in the context of CRNs. IEEE 802.22 WRANs employ contention beacon protocol (CBP) to deal with self co-existence problems. However, these protocols do not take into consideration the fact that these channels can be heterogeneous in the sense that they can vary in their quality such as bandwidth, SNR etc. Without any mechanism that enforces fairness in accessing varying quality channels, coexistence for CRNs is likely to become a very difficult task. In this paper, we model heterogeneous spectrum sharing in CRNs as a non-cooperative evolutionary game where the payoff for every player in the game is determined by the quality of the spectrum band to which it is able to gain access.

Motivation: The FCC requires CRNs to periodically access online databases for up-to-date information about PU activity in their area of operation [1], [3]. Information regarding availability of spectrum bands for secondary access in a given region can therefore be assumed as common knowledge and the amount of PU activity which for the purpose of this paper also determines a channel's quality, can be observed / learnt over a period of time. Notice however, that any other metric can be used to represent a channel's quality without affecting subsequent analysis. Being rational about their choices, every player has a clear preference of selecting the best available channels with the lowest probability of PU activity before the start of every time slot. However, if every player tries to access the best available channel, it will result in collision and the spectrum opportunity being wasted. Players that eventually gain access to channels with low probability of PU activity will gain higher payoffs as compared to the players that end up accessing higher PU activity channels. To study how players (CRNs) in the spectrum sharing game evolve their channel selection strategies, we formulate an evolutionary game theoretic framework and prove that the mixed strategy Nash Equilibrium is the evolutionarily stable strategy (ESS). We also derive the underlying process of Replicator Dynamics by which players evolve their strategies over a period of time and converge to ESS. Adaptation of channel selection strategies is based on CRNs' payoffs at every stage of the game i.e., every time slot. To the best of our knowledge, this research is the first attempt to solve the problem of heterogeneous spectrum sharing in CRNs with the help of evolutionary game theoretic concepts, however some of the related works are presented next.

Evolutionary game is applied in [4] to solve the problem in a joint context of spectrum sensing and sharing within a single CRN. Multiple SUs are assumed to be competing for unlicensed access to a single channel. SUs are considered to have half-duplex devices so they cannot sense the channel and access it at the same time. The problem of self-coexistence in CRNs is dealt with in [5] using a graph theoretic approach called utility graph coloring (UGC). Allocation of spectrum for multiple overlapping CRNs is done in UGC to minimize interference and maximize spectrum. Evolutionary game theoretic concepts are applied in [6] to make secondary users (SU) of a CRN participate in collaborative spectrum sensing in a decentralized manner. The authors of [7] tackle the self-coexistence problem of finding a mechanism to achieve the minimum number of wasted time slots for every collocated CRN to find an empty spectrum band for communications. They employ a distributed modified minority game under incomplete information assumption. Punishment strategies in a Gaussian Interference Game (GIG) are employed in [8] for a one shot game as well as an infinite horizon repeated game to enforce cooperation in spectrum sharing among SUs of a CRN.

II. SYSTEM MODEL

In this paper, we consider a region where overlapping CRNs co-exist and compete with each other for secondary access to the licensed spectrum bands. We model the entire spectrum band that is available

for unlicensed use by CRNs as a set of $K = (1, 2, ..., k)$ channels. The spectrum band is heterogeneous by virtue of the quality of a channel which is determined by the probability P_k with which PUs access their licensed channels. Higher P_k for a given channel k means it is of a lower quality and vice versa and CRNs compete to access the best quality channels. Gaining access to higher quality channel results in higher payoff while lower quality channel yields lower payoff for CRNs where payoff $u_k = 1 - P_k$ from gaining access to channel k. CRNs need to gain access to a channel in every time slot also called a Channel Detection Time (CDT) slot [2]. CRNs are independent entities i.e., they do not share a common goal and therefore do not cooperate with each other. It is in every CRN's interest to gain access to the channels with minimum PU activity i.e., minimum value of P_k. When two or more CRNs select the same channel for access in a given time slot, a contention situation arises and that particular time slot's spectrum opportunity is wasted. Since knowledge of PU's spectrum allocation/activity is mandated by the FCC for CRNs [1], [2], is publically available through online databases and also sensed by CRNs at regular intervals, calculation of the current values of P_k by CRNs is trivial. Having payoffs for selecting a specific channel derived from common knowledge such as P_k is an intuitive choice and makes distributed implementation of our proposed framework possible. As demonstrated subsequently, the number of collocated CRNs does not play any part in the game model because an evolutionary game is concerned with the evolution of *strategies*, associated payoffs and their stability.

III. Evolutionary Spectrum Sharing Game

When strategic interactions of a population mix result in convergence to equilibrium state over a period of time and if that state cannot be invaded by a mutant strategy then the incumbent strategy is called Evolutionarily Stable Strategy (ESS). The objective of an evolutionary game is to evolve a strategy that is ESS. Using this concept, we model the problem of self-coexistence and heterogeneous spectrum sharing in the following subsections as an evolutionary game framework.

A. Game Formulation

The evolutionary game is represented as $\langle C, (A), (U) \rangle$, where players in the game are CRNs represented by C. Let $K = \{1, 2, ..., k\}$ denote the set of available channels for secondary access. Every player in the game has the same action space represented by the set $A = \{a_1, a_2, ..., a_k\}$ where the strategy a_k means selecting channel k for communication during the next time. Also, there is a bijection between the sets A and K. The CRNs gain a specific payoff when they are successful in utilizing a spectrum opportunity in a channel. The payoff for players playing strategies a_k and a_j when competing against each other is denoted by the ordered pair $u(a_k, a_j) \in U$ and is a function of P_k given by:

$$u(a_k, a_j) = \begin{cases} (1 - P_k, 1 - P_j) \text{ when } k \neq j \\ (0, 0) \text{ when } k = j \end{cases} \quad 0 < P_k, P_j < 1 \tag{1}$$

where the first element of the ordered pair $u(a_k, a_j)$ represents the payoff for players that selected channel k and the second element for players selecting channel j. The aforementioned game can also be represented in strategic form as Table I, which shows how two strategies perform when they are pitched against themselves and against other strategies. Without any loss of generality, assume that $P_k < P_j$, then all CRNs will be tempted to choose channel k instead of channel j for a larger payoff. Also, when $k = j$ i.e., when two or

TABLE I: Strategic form representation of Evolutionary Spectrum sharing game with strategies a_k and a_j.

	a_k	a_j
a_k	$(0, 0)$	$(1 - P_k, 1 - P_j)$
a_j	$(1 - P_j, 1 - P_k)$	$(0, 0)$

more players select the same channel for the upcoming time slot, the payoff is 0 for all players as it results in collision. For the sake of simplicity and without the loss of generality, we show the game formulation and its analyses with two competing strategies i.e., the players select one out of the two available channels. The same mechanism can be employed for multiple heterogeneous channels as shown in the performance evaluation section.

B. Equilibrium strategies for the Evolutionary Spectrum Sharing Game

For the sake of completeness, we provide the definitions of pure and mixed strategy Nash Equilibria (NE) before analyzing the ESS.

Definition 1: The pure strategy Nash Equilibrium of the spectrum sharing game is an action profile $a^* \in A$ of actions, such that [9]:

$$u(a_i^*, a_{-i}^*) \succeq u(a_i, a_{-i}^*), \forall i \in K \tag{2}$$

where $\succeq$ is a preference relation over utilities of strategies a_i^* and a_i. The above definition means that for a_i^* to be a pure strategy NE, it must satisfy the condition that no player i has another strategy that yields a higher payoff than the one for playing a_i^* given that every other player plays their equilibrium strategy a_{-i}^*.

Lemma 1: Strategy pairs (a_k, a_j) and (a_j, a_k) are pure strategy NE of the evolutionary game.

Proof: Consider a pairwise competition of two channel selection strategies employed by two CRNs picked randomly from a large population. Assume player 1 to be the row player and player 2 to be the column player in table I. From (1) it follows that both $(1 - P_k)$ and $(1 - P_j)$ are positive values and therefore the payoffs for strategy pairs (a_k, a_j) and (a_j, a_k) are greater than the payoffs for strategy pairs (a_k, a_k) and (a_j, a_j). Consider the payoff for strategy pair (a_k, a_j) from table I. Given that the player playing strategy a_j continues to play this strategy, then from definition-1 for a Nash Equilibrium, it follows that the player playing strategy a_k does not have any incentive to change its choice to a_j, i.e., it will receive a smaller payoff of 0 if it switched to a_j. Therefore, (a_k, a_j) is a pure strategy NE. Similarly, strategy pair (a_j, a_k) is the second pure strategy Nash Equilibrium for this game.

Definition 2: The mixed strategy Nash Equilibrium of the spectrum sharing game is a probability distribution $\hat{p}$ over the set of actions A for any player such that [9]:

$$\hat{p} = (p_1, p_2, ..., p_k) \in \mathbb{R}_{\geq 0}^k, \text{and} \sum_{j=1}^{k} p_j = 1 \tag{3}$$

which makes the opponents indifferent about the choice of their strategies by making the payoffs from all of their strategies equal. In (3), p_k is the probability of a CRN selecting channel k. This also makes the proportion of population that selects channel k for communication in a given time slot equal to P_k. For a pairwise competition in the CRN population and a $2-$channel scenario, let α be the probability with which player 1 plays strategy a_k and $\beta = (1 - \alpha)$ be the probability of playing strategy a_j, then from

the payoffs of table I, the expected payoff of player 2 for playing strategy a_k is given by:

$$EU_2(a_k) = \alpha u(a_k, a_k) + \beta u(a_j, a_k) = \alpha(0) + \beta(1 - P_k) \quad (4)$$

Similarly, the expected payoff of player 2 for playing strategy a_j is given by:

$$EU_2(a_j) = \alpha u(a_k, a_j) + \beta u(a_j, a_j) = \alpha(1 - P_j) + \beta(0) \quad (5)$$

According to definition 2, player 2 will be indifferent about the choice of strategies when the expected payoffs from playing strategies a_k and a_j are equal, i.e.,

$$EU_2(a_k) = EU_2(a_j) \quad (6)$$

Substituting (4) and (5) in (6), we have $\beta(1 - P_k) = \alpha(1 - P_j)$. Therefore:

$$\alpha = \frac{1 - P_k}{(1 - P_k) + (1 - P_j)} \quad (7)$$

$$\beta = \frac{1 - P_j}{(1 - P_k) + (1 - P_j)} \quad (8)$$

Therefore, the mixed strategy NE for the evolutionary game is given by the distribution $\hat{p} = \{\alpha, \beta\}$ which means that when both players select strategies a_k and a_j with probabilities α and β respectively, then their opponents will be indifferent about the outcomes of the play. This also means that all CRNs in a given region form a polymorphic population in which every CRN mixes for its choice of available channels according to the probability distribution $\hat{p}$ which is the mixed strategy NE for our evolutionary channel sharing game.

C. Analysis of Evolutionary stability of the Equilibria

We analyze the evolutionary stability of the Nash equilibria with the help of definition 3 as follows:

Definition 3: For a strategy to be ESS, it must satisfy the following conditions [10]:

1. $u(\hat{p}, \hat{p}) \geq u(p', \hat{p})$ and
2. if $u(\hat{p}, \hat{p}) = u(p', \hat{p})$ then $u(\hat{p}, p') > u(p', p')$

where $\hat{p}$ is the strategy played by the population and can therefore be termed as the population's incumbent strategy while p' is a mutant strategy that competes with the incumbent strategy. According to the first condition of definition 3, an incumbent strategy (1) must be a symmetric NE and (2) must perform at least as good against itself as it does against a mutant strategy. According to the second condition of definition 3, if an incumbent strategy is not a strict NE then the incumbent strategy must do strictly better against a mutant than a mutant strategy does against itself. Now we analyze both pure and mixed strategy NE according to definition 3 to see if they are evolutionarily stable.

Evolutionary Stability of pure strategy NE: We proved that the strategies (a_k, a_j) and (a_j, a_k) are the pure strategy NE of our game in section III-B. If two players play the same strategy i.e., play $(\hat{p}, \hat{p})$ and are in equilibrium, then it is said to be a symmetric NE. Clearly, the pure strategy NE of our game are not symmetric NE and by equation (1) $u(\hat{p}, \hat{p}) < u(p', \hat{p})$. Therefore, pure strategy NE is not evolutionarily stable according to definition 3. This also shows that a strategy may be an equilibrium point for a one-shot game; however it might not be evolutionarily stable in situations where the game is to be played indefinitely.

Evolutionary Stability of mixed strategy NE: With no pure strategy NE for our evolutionary game as ESS, we now determine if the mixed strategy NE that we derived in equations (7, 8) is an ESS according to definition 3. To do so, we first calculate $u(\hat{p}, \hat{p})$ i.e., see how the incumbent strategy $\hat{p}$ fares against itself and then determine the payoff of a mutant strategy p' against the incumbent strategy. Consider the payoff matrix of table I where the players select strategies a_k and a_j with the probability distribution of the incumbent strategy $\hat{p} = \{\alpha, \beta\}$ then:

$$u(\hat{p}, \hat{p}) = \alpha\beta(1 - P_k) + (1 - P_j) \quad (9)$$

In equation (9) above, we have determined the payoff of incumbent strategy $\hat{p}$ when it competes against itself i.e., $u(\hat{p}, \hat{p})$. Now consider a mutant strategy $p' = \{\alpha + \delta, \beta - \delta\}$ which is greedier than the incumbent strategy $\hat{p}$ and assume that it selects the higher quality channel k with a higher probability i.e., $\alpha + \delta$ and selects the lower quality channel j with lower probability i.e., $\beta - \delta$, where δ is a small positive number that represents the increase in greediness/probability of a mutant strategy to select a higher quality channel. Because of the existence of two competing strategies, we now calculate $u(p', \hat{p})$ i.e., the utility of the mutant strategy against the incumbent strategy:

$$\implies u(p', \hat{p}) = \alpha\beta\{(1 - P_k) + (1 - P_j)\} \\ -\delta\{\alpha(1 - P_k) - \beta(1 - P_j)\} \quad (10)$$

Since $P_k < P_j$ as assumed in section III-A, we know that $\alpha(1 - P_k)$ is greater than $\beta(1 - P_j)$ and therefore the second term of equation (10) is positive. From equations (9) and (10) we have $u(\hat{p}, \hat{p}) > u(p', \hat{p})$. Since $u(\hat{p}, \hat{p})$ is strictly greater than $u(p', \hat{p})$, we do not need to check for the second condition of definition 3 and we conclude that the incumbent strategy $\hat{p}$ does strictly better than the mutation p', which will die out in the evolutionary game. Hence our mixed strategy NE cannot be invaded by the greedier mutation p' and is therefore an ESS.

D. Replicator Dynamics for the Evolutionary Game

So far we have shown that the mixed strategy NE of our proposed evolutionary game framework is evolutionarily stable. Evolutionary stability has provided us with a means to evaluate how the channel selection strategies perform in the long run when the CRNs do not cooperate with each other. This concept is somewhat static in nature because it does not demonstrate the dynamics with which the strategies converge to an equilibrium state. Replicator Dynamics [11] explain how players evolve their behaviors by learning through strategic interactions at every stage/generation of the game to reach the equilibrium state which is evolutionarily stable. Now we derive the Replicator Dynamics of our evolutionary game framework with k channels.

From section III-B, let $\hat{p} = \{p_1, p_2, ..., p_k\}$, where $\sum_{j=1}^{k} p_j = 1$, represent the incumbent strategy of selecting channel k with probability p_k. Furthermore, let u_0 be the initial fitness of every CRN, the average payoff of CRNs selecting channel k at a given stage of the game be represented by the set $U = \{u_1, u_2, ..., u_k\}$ and let $\bar{u}$ be the total payoff of the entire CRN population at any given time. Then payoff for a CRN selecting channel k can be calculated as:

$$u_k = u_0 + \sum_{m=1}^{|\mathcal{K}|} p_k u(a_k, a_j), \forall k, j \in \mathcal{K} \quad (11)$$

where $u(a_k, a_j)$ is the fitness of a CRN that selects channel k in a pairwise competition against a CRN that selects channel j. The

average payoff $\bar{u}$ of the entire population of CRNs is given by:

$$\bar{u} = \sum_{n=1}^{k} p_n u_n, \forall n \in K \tag{12}$$

Then probability p'_k of a CRN selecting channel k for the next stage/time slot of the game is given by:

$$p'_k = p_k + \frac{p_k(u_k - \bar{u})}{\bar{u}} \tag{13}$$

Equations (11)-(13) are the replicator dynamics of our evolutionary spectrum sharing game. The idea behind the replicator dynamics is that if selecting channel k in the current time slot results in a higher average fitness for the CRNs that selected it than the overall fitness of the entire CRN population, then the proportion of CRNs selecting channel k in the next time slot will increase. In general, if selecting a particular channel in a given time slot results in a higher than total average payoff then that channel will be selected more frequently in subsequent time slots.

IV. SIMULATIONS AND RESULTS

A. Simulation Setup

We first show the results of simulations in which the collocated CRNs have only two available channels for which they contend and converge to an evolutionary stable state. Next, we show that our evolutionary game converges to ESS with 5 heterogeneous channels as well. We have carried out simulations for a variety of network conditions such as PU activity levels and available channels but omit their discussion due to space limitation.

B. Simulation Results

Figure 1a shows how CRNs select one out of two available channels during every (CDT [2]) time slot with some probability where without any loss of generality, channel 1 is assumed to be of higher quality than channel 2. These CRNs gain a payoff from such strategic interactions shown in figure 1b and modify the probabilities of selecting the same channels in subsequent time slots based on payoffs. Let us first consider payoffs of CRNs that are less greedy. As shown in figure 1b, CRNs that are less greedy and select the lower quality channel receive a larger average payoff at $t = 1$ than CRNs selecting higher quality channel, which makes them increase the probability of selecting the lower quality channel at $t = 2$ (figure 1a). This however, results in lower average payoff for them at $t = 2$ than at $t = 1$ which is still greater than the weighted sum of average payoffs resulting in even greater probability of selecting lower quality channel in subsequent stage. A similar yet opposite pattern can be seen for CRNs that are greedier and select higher quality channels with higher probabilities. CRNs keep modifying their channel selection probabilities in the same manner until their payoffs converge and they reach the ESS, which in the case of figure 1a is $p_1 = 0.54$ and $p_2 = 0.46$ when payoffs for channels 1 and 2 are $u_1 = 0.9$ and $u_2 = 0.75$ respectively. The average payoff u_k of selecting channels is calculated by having the initial payoff u_0 of equation (11) normalized to 1. Weighted sum of average payoffs for the system reaches its maximum and becomes stable when the channel selection strategies converge to ESS.
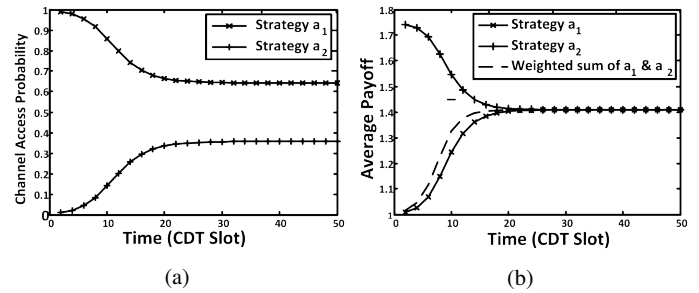

Fig. 1: (a) Probability of selecting channels 1 and 2 i.e., strat a_1 & a_2. (b) Average and weighted sum of Payoffs. Convergence to ESS occurs at $t = 25$ when prob. of selecting strat a_1 approaches 0.54.
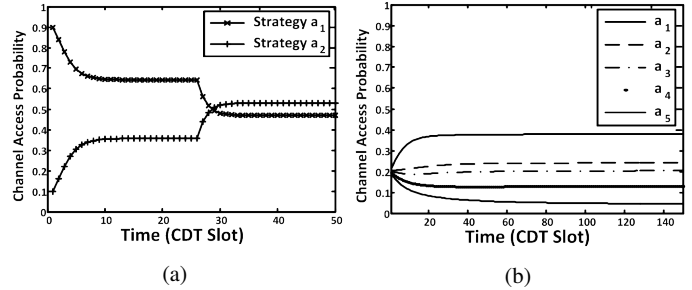

Fig. 2: (a) The game converges to ESS even when network conditions change at $t = 25$ (b) Convergence to equilibrium probabilities for accessing 5 channels of varying quality.

Figure 2a demonstrates the behavior of CRNs playing our proposed evolutionary game under changing network conditions. At the start of simulation, network conditions were initialized at $u_1 = 0.9$ and $u_2 = 0.5$, and after 10 time slots the evolutionary game converged to ESS. Channel conditions were changed to $u_1 = 0.75$ and $u_2 = 0.85$ at $t = 25$ which indicates that channel 1, which was previously of better quality than channel 2, became less attractive for CRNs because its PU became more active on the spectrum. The CRNs playing the evolutionary game were able to adapt to changing network conditions and again converge to a different ESS at $t = 35$. As a result of approaching ESS, all CRNs receive the same payoff and the heterogeneous spectrum resource being distributed fairly.

Figure 2b shows the results of simulating our proposed evolutionary game with 5 channels. It shows that our evolutionary game converges to ESS even with multiple channels of varying quality. However, increase in number of available channels results in slower convergence to ESS at $t = 80$ with 5 channels as compared with $t = 25$ with 2 channels in figure 1a.

V. CONCLUSION

Coexistence protocols employed by CRNs do not consider that spectrum bands vary in *quality* thereby making some channels of the spectrum bands more attractive to CRNs than others. In this paper, we aimed at answering the fundamental question of how CRNs should share heterogeneous spectrum bands fairly in a distributed manner and proposed an evolutionary game theoretic framework to achieve that. We derived equilibrium strategies for CRNs spectrum sharing game for selecting particular spectrum bands and proved that the mixed strategy Nash Equilibria derived in the process are evolutionarily stable strategies (ESS). We also derived the Replicator Dynamics with which players learn from payoff outcomes of their strategic interactions and approach ESS solely upon the common knowledge payoff observations making possible its distributed implementation.

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