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Discrete r.v. X

- $E[X] = \sum k P_X(k)$ $\Rightarrow$

Continous r.v. X

- $E[X] = \int_{-\infty}^{\infty} k f(k) dk$ $\Rightarrow$

Discrete r.v. X:

- $E[g(X)] = \sum g(x)p(x)$

Continuous r.v. X:

- $E[g(X)] = \int_{-\infty}^{\infty} g(x) f(x) dx$

$F_{XY}(x,y)=F_X(x)F_Y(y)$

$P(X \le x, Y \le y) \leftarrow F_{XY}(x,y)$

$P(X \le x) \leftarrow F_X(x)$

$$\underbrace{X \le x}_{A}, \quad \underbrace{Y \le y}_{B}$$

$$P(X \le x, Y \le y) = P(A) \cdot p(B) = P(X \le x) \cdot P(Y \le y) = F_X(x) \cdot F_Y(y)$$

r.v. $X \geq 0$ $\quad E[X] = 10$ $\quad P(X \geq 30)$?

$\leq E[X]/30 = 1/3$

$P(X \geq \alpha) \leq E[X]/\alpha$

$$P(Q) = 1 - P(|X-\mu| \geq k)$$

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$$\geq 1 - (\sigma/k)^2 = 0.75$$

Q: 75% chance of an article being between 760 and 1240 characters long

r.v. $X \sim N(1000, 200^2)$

$\mu = 1000, \ \sigma = 200$

$$p(600 \leq X \leq 1400) = p(|X - 1000| \leq 400)$$

$$= 1 - p(|X - 1000| > 400)$$

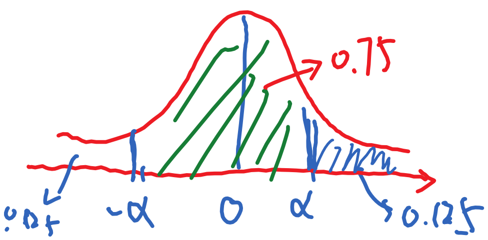
$$\geq 1 - \left(\frac{200}{400}\right)^2 = 0.75$$

$\boxed{Q}$: find Δ, such that $p(|X-\mu| \leq \Delta) = 0.75$.

$\boxed{A}$: r.v. $Z \sim N(0,1)$. $Z = \frac{X - 1000}{200}$, $p(|X-\mu| \leq \Delta) = p(|\sigma Z| \leq \Delta)$

$$= p\left(|Z| \leq \underbrace{\frac{\Delta}{\sigma}}_{\alpha}\right) = 0.75$$

$\hookrightarrow X = \sigma Z + \mu$



$$p(Z \leq \alpha) = 0.75 + 0.125 = 0.875 \rightarrow \alpha = 1.2 \qquad \Delta = 240$$

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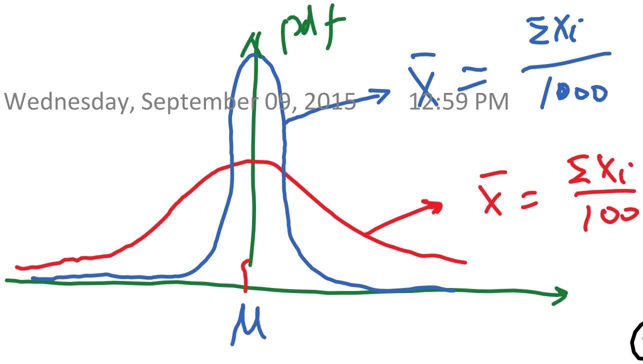

$E[X_i]=0.5$, $Var(X_i)=1/12$

Q: $P(\sum_{i=1}^{10} X_i > 7)$

$$\bar{X} = \sum_{i=1}^{n} X_i/n$$
$$E[\bar{X}] = \mu$$
$$Var(\bar{X}) = \sigma^2/n$$

$$\bar{X} = \frac{\sum X_i}{10}$$

Q: $P(\sum X_i > 7) = P(\bar{X} > 0.7)$

$E[\bar{X}] = 0.5$, $Var(\bar{X}) = \frac{1}{120}$ $\quad \bar{X} \sim N(0.5, \frac{1}{120})$

r.v. $Z = \frac{\bar{X} - 0.5}{\sqrt{\frac{1}{120}}} \sim N(0,1)$

$$P(\bar{X} > 0.7) = P\left(\frac{Z}{\sqrt{120}} > 0.2\right) = P(Z > 2.19)$$
$$= 1 - P(Z \le 2.19) = 1 - 0.986$$