

A blood test is 95% accurate (detects a sick person as sick), but has 1% false positive (detects a healthy person as sick). We know 0.5% population are sick.

Q: if a person is tested positive, what is the prob. she is really sick?

$E: \{ \text{tested positive} \}$

$S: \{ \text{the person is sick} \}$

$$P(E|S) = 0.95$$

$$P(E|\bar{S}) = 0.01$$

$$P(S) = 0.005$$

Q:  $P(S|E)$  ?

$$P(S|E) = \frac{P(SE)}{P(E)} = \frac{P(E|S) \cdot P(S)}{P(E)} = 0.323$$

$$\begin{aligned} P(E) &= P(E|S) \cdot P(S) + P(E|\bar{S}) \cdot P(\bar{S}) \\ &= 0.95 \times 0.005 + 0.01 \times 0.995 \end{aligned}$$

1000 persons  $\Rightarrow$  5 sick, 995 healthy  
 ↓  
 5 positive       $\rightarrow$  10 positive

① Model:

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$$E = \{ \text{dollar, cheap} \}$$

email contains

12:27 PM

$$P(S|E) ?$$

$$P(\cancel{S}|E) ?$$

$$P(\text{dollar}|S) = 0.2$$

$$P(\text{cheap}|S) = 0.5$$

$$P(\text{dollar}|H) = 0.05$$

$$P(\text{cheap}|H) = 0.01$$

$$P(S) = 0.1$$

$$P(H) = 0.9$$

From training data, we know that a spam email has prob. 0.2 to contain 'dollar', 0.5 to contain 'cheap', ....; a normal email has prob. 0.05 to contain 'dollar', 0.01 to contain 'cheap', ....  
Among all received emails by our email server, 10% are spam and 90% are normal emails

$$P(S|E) = \frac{P(SE)}{P(E)} = \frac{P(E|S) \cdot P(S)}{P(E)} = 0.957$$

$$P(E|S) = P(\text{dollar}|S) \cdot P(\text{cheap}|S) = 0.1$$

$$P(E) = P(E|S) \cdot P(S) + P(E|H) \cdot P(H) = \\ 0.05 \downarrow \times 0.01$$

$$f_X(1) = p$$

$$f_X(0) = 1-p$$

$$P(A \cap B) = P(A) \cdot P(B)$$

$$P(m, h) = p^m \cdot p^h = (1-p) \cdot p$$

$$p = 0.1, n = 35$$

r.v.  $X$ : # of active users

$$\begin{aligned} P(\text{congestion}) &= P(X > 10 \text{ users active at the same time}) \\ &= P(X > 10) = P(X=11) + \dots + P(X=35) \end{aligned}$$

$$= 1 - P(X \leq 10) = 1 - [P(X=0) + P(X=1) + \dots + P(X=10)]$$