M/M/1 queue formulas: $\rho = \frac{\lambda}{\mu} < 1$

Monday, November 16, 2015 11:59 AM

$\pi_0 = 1 - \rho$, $E[N] = \frac{\rho}{1-\rho}$

T: sojourn (response) time

$E[T] = \frac{1}{\mu} + E[W] = \frac{1}{\mu - \lambda}$

, $E[W] = E[N] \cdot \frac{1}{\mu}$

$\lambda = 80/sec$ $\mu = 100/sec$ $\rightarrow \rho = \frac{4}{5}$ $\pi_i \equiv p(N=i)$

Q1: $E[N]$? $E[N] = \frac{0.8}{0.2} = 4$

Q2: $E[W]$? $E[W] = E[N] \cdot \frac{1}{\mu} = 4 \times \frac{1}{100} = 0.04$ sec

Q3: $E[T]$? $E[T] = \frac{1}{\mu - \lambda} = \frac{1}{100-80} = \frac{1}{20} = 0.05$ sec

$\pi_n = \rho^n \pi_0$

Q4: π_0? $\pi_0 = 1 - \rho = 0.2$

Q5: $p(N>5) = 1 - p(N \le 5) = 1 - p(N=0) - p(N=1) - \dots p(N=5) = 1 - \pi_0 - \pi_1 - \dots - \pi_5$

Q6: μ? such that $E[T] = 0.02$ sec ?

$E[T] = \frac{1}{\mu - \lambda} = \frac{1}{\mu - 80} = 0.02 \Rightarrow \mu - 80 = 50 \Rightarrow \mu = 130$

$\Rightarrow$ bandwidth should be 130 kbps M/G/1

$$\frac{\lambda^k x^{k-1} e^{-\lambda x}}{(k-1)!}$$ ← kth Erlang pdf rate λ

Monday, November 16, 2015 12:33 PM

$$f_{T|N}(t|n) = \frac{\mu(\mu t)^n e^{-\mu t}}{n!} \Leftarrow \frac{\mu^{n+1} t^n e^{-\mu t}}{n!}$$

$$e^{\alpha} = \sum_{n=0}^{\infty} \frac{\alpha^n}{n!}$$

↓

(n+1)-order Erlang rate μ

exp. λ $f(t) = \lambda e^{-\lambda t}$

$$f_T(t) = \sum_{n=0}^{\infty} (1-\rho)\rho^n \frac{\mu(\mu t)^n e^{-\mu t}}{n!} = (\mu-\lambda) e^{-(\mu-\lambda)t} = \lambda' e^{-\lambda' t}$$

$\pi_i = \rho^i \pi_0$ $f_{T|N}(t|N=n)$

$\pi_0 = 1-\rho$

$\mu-\lambda$ → λ' $\quad (\mu-\lambda)t$ → $\lambda' t$

$X_1, X_2, X_3 \sim$ expo. distr. μ $\quad P(X > t) = e^{-\mu t}$

$P(X \le t) = 1 - e^{-\mu t}$

Monday, November 16, 2015 1:10 PM

(3) $\xrightarrow{3\mu}$ (2) jump time : $Y = \min(X_1, X_2, X_3)$

$$P(Y \le t) = 1 - P(Y > t) = 1 - P(X_1 > t, X_2 > t, X_3 > t)$$

$$= 1 - P(X_1 > t) \cdot P(X_2 > t) \cdot P(X_3 > t)$$

$$= 1 - e^{-3\mu t}$$

$\Rightarrow$ Y also expo. distr. with rate 3μ