S.I. example: two basketball teams play indoor game, and the outside weather

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Sun, rain

Home team win, lose

$S = \{(s,w), (s,l), (r,w), (r,l)\}$

$A = \{\text{Home team Win}\}$ — $(s,w), (r,w)$, $B = \{\text{weather is sunny}\}$ — $(s,w), (s,l)$

A and B S.I.

$C = \{\text{Home team win}\}$ — $(s,w), (r,w)$ $D = \{\text{guest team win}\}$ — $(s,l), (r,l)$

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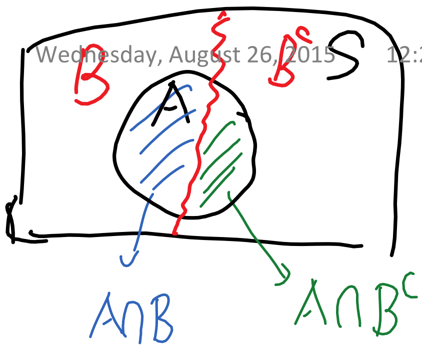

$A = AB \cup AB^c$ mutually exclusive

$P(A) = P(AB) + P(AB^c)$ ← from ground truth

$= P(A|B)P(B) + P(A|B^c)P(B^c)$

$P(A|B) = P(AB)/P(B)$

A man shoots an outdoor target. When sunny day, he has 0.8 prob. to hit the target; when raining day, he has 0.4 prob. to hit. The weather has 0.7 prob. to be sunny, and 0.3 prob. to be raining.

Q: $P(A)$? $A = \{$he hits target today$\}$

① Modeling: S: Sunny $P(S) = 0.7$

R: today is raining $P(R) = 0.3$

$P(A|S) = 0.8$, $P(A|R) = 0.4$

② Computing: $P(A) = P(A|S) \cdot P(S) + P(A|R) \cdot P(R)$

$= 0.8 \times 0.7 + 0.4 \times 0.3 = 0.68$

two-pick game:

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① Modeling:

$S = \{$ stick to first pick and win $\}$

$C = \{$ exchange in second and win $\}$

$Q: P(S) \overset{?}{>} P(C)$

$R_1 \{$ first picked signed card $\}$ $P(R_1) = 1/3$ $P(L_1) = 2/3$

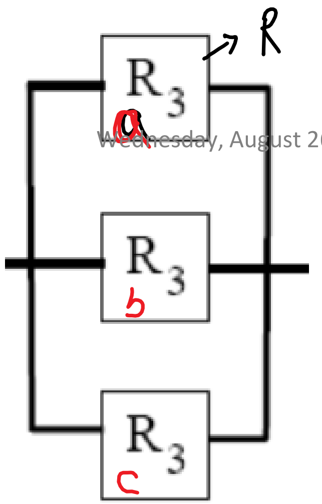
$L_1 \{$ first picked blank card $\}$

② Computing:

$$P(S) = P(S|R_1) \cdot P(R_1) + \underbrace{P(S|L_1)}_{=0} \cdot P(L_1) = P(R_1) = 1/3$$

$$P(C) = P(C|R_1) \cdot P(R_1) + P(C|L_1) \cdot P(L_1) = 0 \cdot P(R_1) + 1 \times P(L_1)$$

$$= 2/3$$



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$$P(\text{system works}) = P(\text{at least one component works})$$

$$= 1 - P(\text{all three fail})$$

$$= 1 - P(a\text{ fails}) \cdot P(b\text{ fails}) \cdot P(c\text{ fails})$$

$$= 1 - (1-R)^3$$

$$P(A|B) = P(AB)/P(B) \Rightarrow P(AB) = P(A|B) \cdot P(B)$$

$$= P(B|A) \cdot P(A)$$

$$= \frac{P(B|A) \cdot P(A)}{P(B)}$$

$$P(B) = P(B|A) \cdot P(A) + P(B|A^c) \cdot P(A^c)$$

$$= \frac{P(B|A) \cdot P(A)}{P(B|A) \cdot P(A) + P(B|A^c) \cdot P(A^c)}$$

A man shoots a target. When sunny day, he has 0.8 prob. to hit the target; when raining day, he has 0.4 prob. to hit. The weather has 0.7 prob. to be sunny, and 0.3 prob. to be raining.

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① Modeling:

H : { hit target }

$\bar{H}$: { miss " }

R : { raining today }

$\bar{R}$: { sunny }

Q: $P(R \mid \bar{H})$?

$P(H \mid \bar{R}) = 0.8$, $P(H \mid R) = 0.4$

$1 - P(\bar{H} \mid \bar{R})$

$P(\bar{R}) = 0.7$, $P(R) = 0.3$

$$P(R \mid \bar{H}) = \frac{P(R\bar{H})}{P(\bar{H})} = \frac{P(\bar{H} \mid R) \cdot P(R)}{P(\bar{H} \mid R) \cdot P(R) + P(\bar{H} \mid \bar{R}) \cdot P(\bar{R})} = \frac{0.6 \times 0.3}{0.6 \times 0.3 + 0.2 \times 0.7} \approx 0.56$$