UCF Stands For Opportunity

CDA6530: Performance Models of Computers and Networks

Chapter 8: Statistical Simulation --- Discrete-Time Simulation

Simulation Studies

- Models with analytical formulas
  - Calculate the numerical solutions
    - Differential equations ---- Matlab Simulink
      - Or directly solve if has closed formula solutions
    - Discrete equations --- program code to solve
  - The mean value formulas for stochastic events
    - Solutions are only for the mean values
  - If you derive models in your paper, you must use real simulation to verify that your analytical formulas are accurate

Simulation Studies

- Models without analytical formulas
  - Monte Carlo simulation
    - Generate a large number of random samples
    - Aggregate all samples to generate final result
    - Example: use U(0,1) to compute integral
  - Discrete-time simulation
    - Divide time into many small steps
    - Update system states step-by-step
    - Approximate, assume system unchanged during a time step
  - Discrete event simulation (DES)
    - Accurate
    - Event-driven

Discrete-Time Simulation

- System is assumed to change only at each discrete time tick
  - Smaller time tick, more accurate simulation for a continuous-time physical system
  - At time k, all nodes' status are only affected by system status at k-1
- Why use it?
  - Simpler than DES to code and understand
  - Fast, if system states change very quickly (or many events happening in short time period)

Discrete-Time Simulation

While (simulation not complete){

1). Time tick: k ++;

2). For system's node i (i=1,2,⋯)

3). Simulate what could happen for node i during the last time step (k-1 → k) based on all nodes status at k-1

4). Update the state of node i if something happens to it

5). Output time tick k's system's states (e.g., status of every node in the system)

}

Discrete-Time Simulation

- Note: when computing system node i's state at time tick k, it should be determined only by all other system nodes' states at time tick k-1
  - Be careful in step 4): DO NOT use node j's newly updated value at current round
    - Newly updated value represents state at the beginning of next round.

Discrete-Time Simulation

- An example: one line of nodes
  - $X_i(t) = (U-0.5) + (X_{i-1}(t-1) + X_{i+1}(t-1)) / 2$

```
Simul_N = 1000; n=100; X = ones(n,1);
for k=1:Simul_N,
    U = rand(n,1);
    X(1) = (U(1) - 0.5) + X(2);
    for i=2:n-1,
            X(i) = (U(i) - 0.5) + (X(i-1) + X(i+1)) / 2;
    end
    X(n) = (U(n) - 0.5) + X(n-1);
    % display or save X value for time k
end
```

What's Wrong?

Discrete-Time Simulation

- Corrected Code:

```
Simul_N = 1000; n=100; X = ones(n,1);
Prior_X = ones(n,1);
for t=1:Simul_N,
     U = rand(n,1);
      Prior_X = X;  /* save last time's data */
     X(1) = (U(1) - 0.5) + Prior_X(2);
     for i=2:n-1,
              X(i) = (U(i) - 0.5) + (Prior_X(i-1) + Prior_X(i+1)) / 2;
     end
     X(n) = (U(n) - 0.5) + Prior_X(n-1);
     % display or save X value for time k
end
```

- **Another way to do the correct coding:**

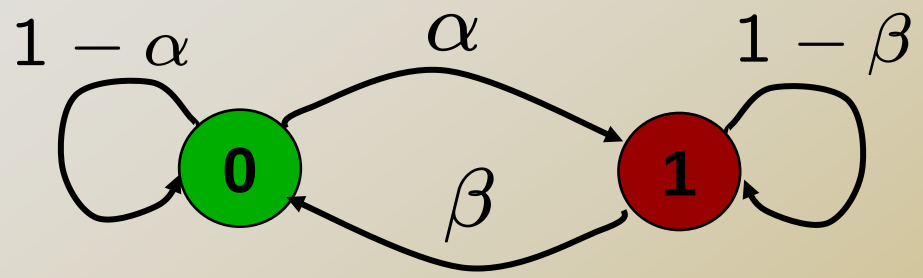
- Simul_N = 1000; n=100; X = ones(n,Simul_N);

```
% X(i, t) is the value of node i at time t.
for t=2:Simul_N,
        U = rand(n,1);
        X(1, t) = (U(1) - 0.5) + X(2,t-1);
        for i=2:n-1,
                X(i,t) = (U(i) - 0.5) + (X(i-1, t-1) + X(i+1,t-1)) / 2;
    end
    X(n,t) = (U(n) - 0.5) + X(n-1, t-1);
    % display or save X value for time k
end
```

Example: Discrete-Time Markov Chain Simulation

- Simulate N steps
- For each step, use random number U to determine which state to jump to
  - Similar to discrete r.v. generation
- $\pi(i) = m_i/N$
  - N: # of simulated steps
  - m_i: number of steps when the system stays in state i.

Discrete-time Markov Chain Example



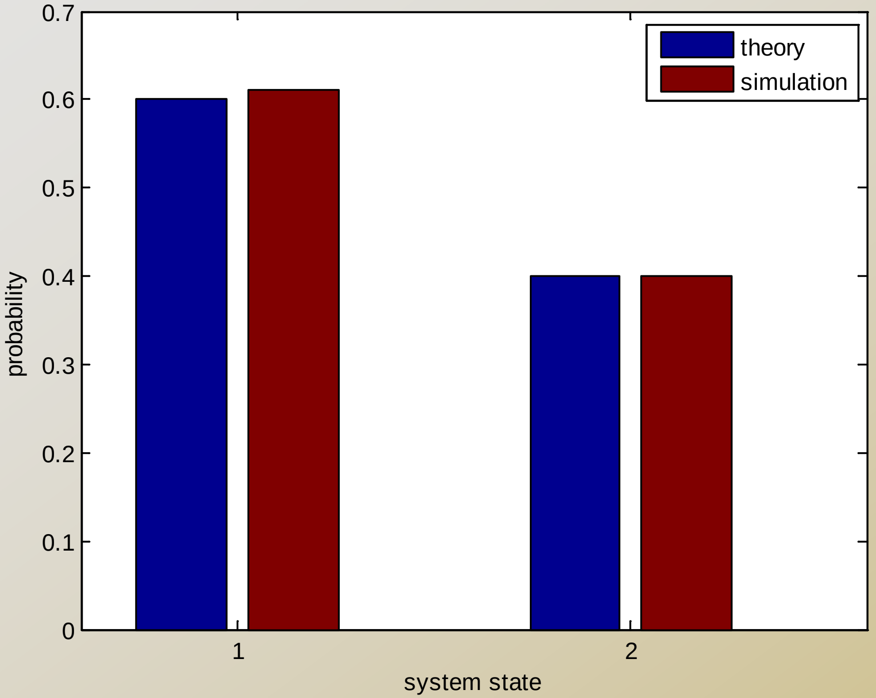
- **Markov on-off model (or 0-1 model)**
- **Q: the steady-state prob.?**

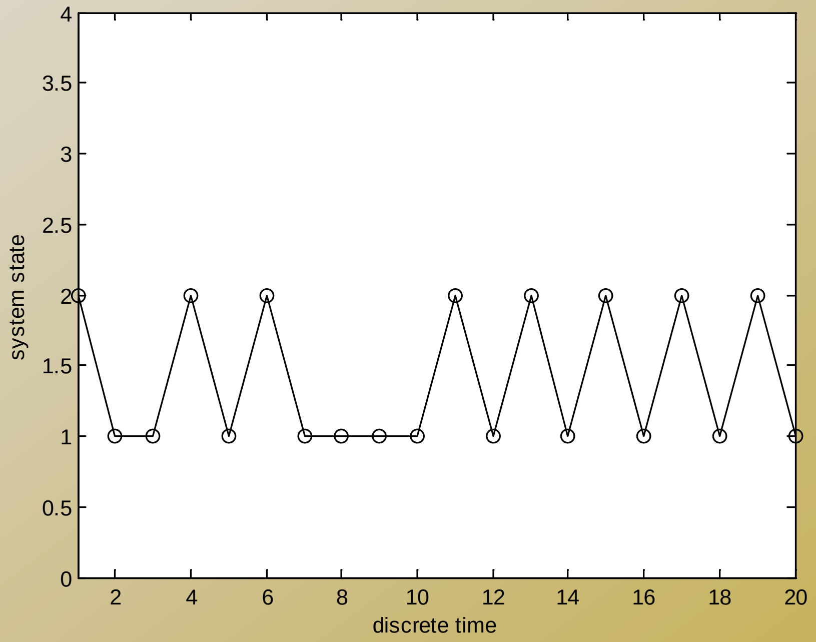
$$\mathbf{P} = \begin{bmatrix} 1-\alpha & \alpha \\ \beta & 1-\beta \end{bmatrix}$$

$$\begin{cases} \pi_0 = & (1-\alpha)\pi_0 + \beta\pi_1 \\ \pi_1 = & \alpha\pi_0 + (1-\beta)\pi_1 \end{cases} \Rightarrow \begin{cases} \pi_0 & = \frac{\beta}{\alpha+\beta} \\ \pi_1 & = \frac{\alpha}{\alpha+\beta} \end{cases}$$

$$\pi_0 + \pi_1 = 1$$

Simulation result (100 time steps)





- bar([Pi_theory Pi_simulation]);
- Pi_theory and Pi_simulation are column vectors

Appendix: Continuous R.V. simulation

- Use inverse transform method:
  - One value of U → one r.v. sample
- Normal distr. use the polar method to generate
- How to draw CDF?
  - Problem: r.v. x could be any value
  - Solve: determine x_i points to draw with fixed interval (i=1,2,…)
  - $F(x_i) = P(X \leq x_i) = m/n$
    - n: # of samples generated
    - m: # of sample values $\leq x_i$

Continuous R.V.

- How to draw pdf (probability density function)?
  - In Matlab, use histc() and bar()
    - N = histc(Y, Edge) for vector Y, counts the number of values in Y that fall between the elements in the Edge vector (which must contain monotonically non-decreasing values). N is a length(Edge) vector containing these counts.
    - Use bar(Edge,N, ‘histc’) to plot the curve
  - The curve plot will have the same curve pattern as f(x), but not the same Y-axis values

Pdf example of continuous R.V.

```
% exponential distribution pdf
lambda = 2; sampleN = 1000;
Sample = zeros(1, sampleN);
U = rand(1, sampleN);
for i=1:sampleN,
   Sample(i) = -log(1-U(i))/lambda;
end
Edge = 0:0.1:5;
N = histc(Sample, Edge);
bar(Edge, N, 'histc');
```