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$f(h) = ch$

$\lim_{h \to 0} \frac{f(h)}{h} = \lim_{h \to 0} c = c$

$$\lim_{h \to 0} \frac{f(h)}{h} = 0$$

$$\begin{aligned} P[\underbrace{X \le t+h}_{A} \mid \underbrace{X > t}_{B}] &= P[X \le h] \quad \textbf{Why?} \\ &= 1 - e^{-\lambda h} \\ &= 1 - [1 - \lambda h + \sum_{n=2}^{\infty} \frac{(\lambda h)^n}{n!}] \\ &= \lambda h + o(h) \end{aligned}$$

$f(x=t) = e^{-\lambda t}$

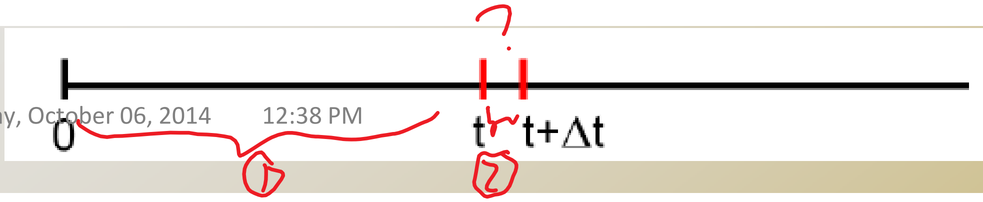
$P(A|B) = \frac{P(AB)}{P(B)}$

$= \frac{P(t < X \le t+h)}{P(X > t)} \Rightarrow \int_t^{t+h} e^{-\lambda t} dt$

$P(X > t) \to e^{-\lambda t}$

$= 1 - e^{-\lambda h}$

$P_n(t) = P(N(t)=n)$



$P_n(t+\Delta t) \longrightarrow P(N(t+\Delta t)=n)$

① $n-1$ arrived by t, 1 arrived in $[t, t+\Delta t]$

② n arrived by t, no arrival in $[t, t+\Delta t]$

$$
\begin{aligned}
P_n(t+\Delta t) &= P_{n-1}(t)\lambda\Delta t + P_n(t)(1-\lambda\Delta t) + o(\Delta t) \\
P_n(t+\Delta t) - P_n(t) &= P_{n-1}(t)\lambda\Delta t - P_n(t)\lambda\Delta t + o(\Delta t) \\
\frac{P_n(t+\Delta t) - P_n(t)}{\Delta t} &= P_{n-1}(t)\lambda - P_n(t)\lambda + \frac{o(\Delta t)}{\Delta t}
\end{aligned}
$$

$$P(N(t+\Delta t)=n) = \underbrace{P(N(t)=n-1)}_{P_{n-1}(t)} \cdot \underbrace{P(N(\Delta t)=1)}_{\lambda \Delta t} + \underbrace{P(N(t)=n)}_{P_n(t)} \cdot \underbrace{P(N(\Delta t)=0)}_{1-\lambda\Delta t}$$

$$\frac{dP_n}{dt} = \lambda P_{n-1} - \lambda P_n \xrightarrow{n=1} \frac{dP_1}{dt} = \lambda P_0 - \lambda P_1 \longleftarrow P_0(t) = e^{-\lambda t}$$

$P_1(t) = \ldots$

$$P_0(t+\Delta t) = P_0(t)(1-\lambda\Delta t)+o(\Delta t)$$

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$P_0(t)$ $\qquad$ $P_0(0)=1$

$\hookrightarrow P(N(0)=0)=1$

$\downarrow$

$P(N(t)=0)$

$X \sim \exp.\ \lambda$

$P(X>t)=e^{-\lambda t}$

$$P_0(t+\Delta t)-P_0(t) = -\lambda P_0(t)\Delta t + o(\Delta t)$$

$$\frac{P_0(t+\Delta t)-P_0(t)}{\Delta t} = -\lambda P_0(t) + \frac{o(\Delta t)}{\Delta t}$$

$$\frac{dP_0'}{dt} = -\lambda P_0 \qquad \underline{P_0(t) = c\cdot e^{-\lambda t}}$$

because $P_0(0)=1$

$\Rightarrow C=1$