assume that, we know $p(X\le t)$

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$Y= \max(X_1, X_2, \cdots, X_n)$
$Z= \min(X_1, X_2, \cdots, X_n)$

Q_1: $p(Z\le t)$ Q_2: $p(Y\le t)$ $p(X\le t)=1-e^{-\lambda t}$

$$p(Z>t) = p(X_1>t, X_2>t, \cdots X_n>t)$$
$$= p(X_1>t)\cdot p(X_2>t)\cdot \cdots \cdot p(X_n>t)$$
$$= [1-p(X_1\le t)]\cdot[1-p(X_2\le t)]\cdots[1-p(X_3\le t)]$$
$$= [1-p(X\le t)]^n$$

Q_1: $p(Z\le t) = 1-[1-p(X\le t)]^n$

$$p(Y\le t) = p(X_1\le t, X_2\le t, X_3\le t \cdots) = p(X_1\le t)\cdot p(X_2\le t)\cdots p(X_3\le t)$$
$$= p(X\le t)^n$$

$\underbrace{p(Y\le t)= (1-e^{-\lambda t})^n}_{\text{if } X\sim \text{exp. distr, then}}$

r.v. X_1 X_2 they are not identical distr.

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$Z = \min(X_1, X_2)$

$p(Z \le t) = 1 - [1 - p(X_1 \le t)] \cdot [1 - p(X_2 \le t)]$

$$\underbrace{0\,0\,0\,1}_{Y_1}\ \underbrace{0\,0\,0\,0\,1}_{Y_2}\ \underbrace{0\,0\,1}_{Y_3}\ \underbrace{0\,1}_{Y_4}$$

$\begin{cases} 1, 0\ 1 \\ 0, 1, 1 \end{cases}$

$Y_i \sim$ Geometric distr. p

r.v. N : # of flips

r.v. Y_i : # of flips until one head

$p(N = n) = p(Y_1 \ge 3, Y_2 \ge 3, Y_3 \ge 3 \dots Y_{n-1} \ge 3, Y_n \le 2)$

$= p(Y_1 \ge 3) \cdot p(Y_2 \ge 3) \cdot \cdots p(Y_n \le 2)$

$p(Y \ge 3) = 1 - p(Y \le 2) = 1 - 2p + p^2 = (1-p)^2$

$p(Y \le 2) = p(Y=1) + p(Y=2) = p + (1-p) \cdot p = 2p - p^2$

$$P(N=n) = (2p-p^2)\cdot(1-p)^{2n-2}$$

$$E[N] = \sum_{n=2}^{\infty} p(N=n)\cdot n = p(2-p)\cdot\sum_{n=2}^{\infty} n(1-p)^{2(n-1)}$$

define $\alpha = (1-p)^2$

$$E[N] = p(2-p)\cdot\sum_{n=2}^{\infty} n\cdot\alpha^{n-1}$$

define $S = \sum_{n=2}^{\infty} n\alpha^{n-1}$

$$S = 2\alpha + 3\alpha^2 + 4\alpha^3 + 5\alpha^4 \cdots$$

$$\alpha S = 2\alpha^2 + 3\alpha^3 + 4\alpha^4 + 5\alpha^5 \cdots$$

$$(1-\alpha)S = 2\alpha + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5 \cdots$$

$$= \alpha + [\alpha + \alpha^2 + \alpha^3 + \alpha^4 \cdots]$$

$$= \alpha + \frac{\alpha}{1-\alpha} \Rightarrow S = \frac{2\alpha - \alpha^2}{(1-\alpha)^2}$$

$$E[N] = p(2-p)\cdot S = \frac{(1-p)^2[2-(1-p)^2]}{(2-p)p}$$

$P(N_t = n)$ $\Rightarrow$ by time t, we hit target n times

tried t times