

assumed, we know

$$p(X \leq t)$$

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$$Y = \max(X_1, X_2, \dots, X_n)$$
$$Z = \min(X_1, X_2, \dots, X_n)$$

$$Q_1: P(Z \leq t) \quad Q_2: P(Y \leq t) \quad p(X \leq t) = 1 - e^{-\lambda t}$$

$$\begin{aligned} P(Z > t) &= P(X_1 > t, X_2 > t, \dots, X_n > t) \\ &= P(X_1 > t) \cdot P(X_2 > t) \cdot \dots \cdot P(X_n > t) \\ &= [1 - P(X_1 \leq t)] \cdot [1 - P(X_2 \leq t)] \cdot \dots \cdot [1 - P(X_n \leq t)] \\ &= [1 - p(X \leq t)]^n \end{aligned}$$

$$Q_1: P(Z \leq t) = 1 - [1 - p(X \leq t)]^n$$

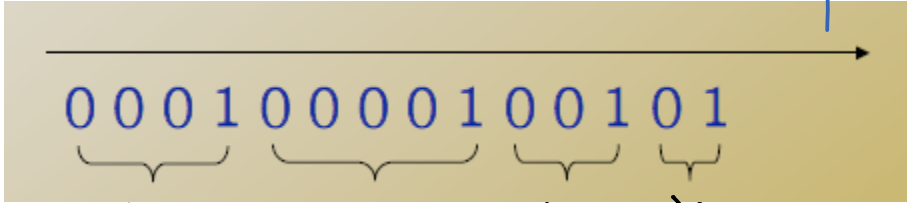
$$\begin{aligned} P(Y \leq t) &= P(X_1 \leq t, X_2 \leq t, X_3 \leq t, \dots) = P(X_1 \leq t) \cdot P(X_2 \leq t) \cdot \dots \cdot P(X_n \leq t) \\ &= p(X \leq t)^n \end{aligned}$$

$\underbrace{p(Y \leq t) = (1 - e^{-\lambda t})^n}_{\text{if } X \sim \text{exp. distr. then}}$

v.v. X_1, X_2 they are not identical distr.

$$Z = \min(X_1, X_2)$$

$$P(Z \leq t) = 1 - [1 - P(X_1 \leq t)] \cdot [1 - P(X_2 \leq t)]$$



Y_1 Y_2 Y_3 Y_4

v.v. N : # of flips

v.v. Y_i : # of flips until one head

$$P(N = n) = P(Y_1 \geq 3, Y_2 \geq 3, Y_3 \geq 3, \dots, Y_{n-1} \geq 3, Y_n \leq 2)$$

$$= P(Y_1 \geq 3) \cdot P(Y_2 \geq 3) \cdot \dots \cdot P(Y_n \leq 2)$$

$$P(Y > 3) = 1 - P(Y \leq 2) = 1 - 2p + p^2 = (1-p)^2$$

$$P(Y \leq 2) = P(Y=1) + P(Y=2) = p + (1-p) \cdot p = 2p - p^2$$

$$\begin{cases} 1, 0, 1 \\ 0, 1, 1 \end{cases}$$

$Y_i \sim$ Geometric distr. p

$$P(N=n) = (2p-p^2) \cdot (1-p)^{2n-2}$$

$$E[N] = \sum_{n=2}^{\infty} P(N=n) \cdot n = p(2-p) \cdot \sum_{n=2}^{\infty} n(1-p)^{2(n-1)}$$

define $\alpha = (1-p)^2$ $E[N] = p(2-p) \cdot \underbrace{\sum_{n=2}^{\infty} n \cdot \alpha^{n-1}}$

define $S = \sum_{n=2}^{\infty} n \alpha^{n-1}$

$$\left\{ \begin{aligned} S &= 2\alpha + 3\alpha^2 + 4\alpha^3 + 5\alpha^4 + \dots \\ \alpha S &= 2\alpha^2 + 3\alpha^3 + 4\alpha^4 + 5\alpha^5 + \dots \end{aligned} \right.$$

$$\begin{aligned} (1-\alpha)S &= 2\alpha + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5 + \dots \\ &= \alpha + [\alpha + \alpha^2 + \alpha^3 + \alpha^4 + \dots] \end{aligned}$$

$$E[N] = p(2-p) \cdot S = \frac{(1-p)^2 [2 - (1-p)^2]}{(2-p)p}$$

$$= \alpha + \frac{\alpha}{1-\alpha} \Rightarrow S = \frac{2\alpha - \alpha^2}{(1-\alpha)^2}$$

$P(N_t = n) \Rightarrow$ by time t , we hit target n times
tried t times