$$
\begin{aligned}
P_{ij}^{n+m} &= P(X_{n+m} = j \mid \underline{X_0 = i}), \\
&= \sum_{k=0}^{\infty} P(\underbrace{X_{n+m} = j}_{A}, \underbrace{X_n = k}_{B} \mid X_0 = i), \quad \text{Why?} \\
&= \sum_{k=0}^{\infty} P(\underbrace{X_{n+m} = j}_{A} \mid \underbrace{X_n = k, X_0 = i}_{B}) P(\underbrace{X_n = k}_{B} \mid X_0 = i), \\
&= \sum_{k=0}^{\infty} P(X_{n+m} = j \mid X_n = k) P(X_n = k \mid X_0 = i), \quad \text{Why?} \\
&= \sum_{k=0}^{\infty} P_{kj}^{m} P_{ik}^{n}
\end{aligned}
$$

$p(AB)$

10/3/2013

$\rightarrow = p(A|B) \cdot p(B)$

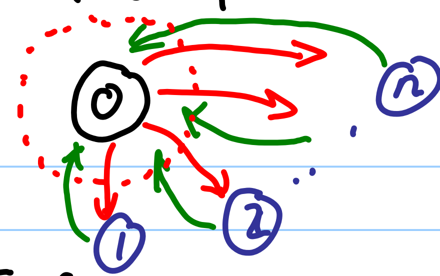
$$
\mathbf{P} = \begin{bmatrix} 1-\alpha & \alpha \\ \beta & 1-\beta \end{bmatrix}
$$

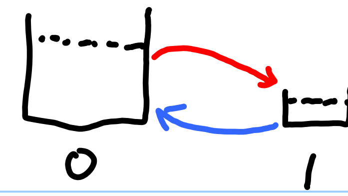
$\pi = \pi P \qquad \pi = [\pi_0 \quad \pi_1]$

$$
[\pi_0 \quad \pi_1] = (\pi_0 \quad \pi_1) \cdot \begin{bmatrix} 1-\alpha & \alpha \\ \beta & 1-\beta \end{bmatrix}
$$

$$\begin{aligned} \pi &= \pi P, \\ \pi\mathbf{1} &= 1 \end{aligned}$$

why? $\pi = (\pi_0 \ \ \pi_1 \ \cdots \ \pi_n)$





→ $\pi_0(P_{01} + P_{02} + \cdots P_{0n}) = \pi_0(1 - P_{00}) = \pi_1 P_{10} + \pi_2 P_{20} + \cdots + \pi_n P_{n0}$

← $= \pi_1 P_{10} + \pi_2 P_{20} + \cdots \pi_n P_{n0}$

$$\pi_0 = \pi_0 P_{00} + \pi_1 P_{10} + \pi_2 P_{20} + \cdots \pi_n P_{n0}$$

$$\pi_1 = \pi_0 P_{01} + \pi_1 P_{11} + \pi_2 P_{21} + \cdots \pi_n P_{n1}$$

$$\vdots \qquad\qquad \vdots$$

$$\boldsymbol{\pi} = \boldsymbol{\pi} \cdot \mathbf{P}$$

$\pi P = \pi$, $\pi\,[1\ 1\ 1 \ldots 1]^T = 1$

3-state MC

$\pi(P-I)=0$

$$(\pi_0\ \pi_1\ \pi_2)\cdot \underbrace{\begin{bmatrix} P_{00}-1 & P_{01} & P_{02} \\ P_{10} & P_{11}-1 & P_{12} \\ P_{20} & P_{21} & P_{22}-1 \end{bmatrix}}_{A} = [\cancel{0}\ 0\ 0]$$

$$(\pi_0\ \pi_1\ \pi_2)\cdot\begin{bmatrix}1\\1\\1\end{bmatrix}=1$$

$\pi \cdot A = [1\ 0\ 0]$

$\pi = [1\ 0\ 0]\cdot A^{-1}$

$$(\pi_0\ \pi_1\ \pi_2)\cdot\begin{bmatrix} P_{00} & P_{01} & P_{02} \\ P_{10} & P_{11} & P_{12} \\ P_{20} & P_{21} & P_{22} \end{bmatrix} - (\pi_0\ \pi_1\ \pi_2)=0$$

$$(\pi_0\ \pi_1\ \pi_2)\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$\pi_0 P_{01} + \pi_1 P_{11} + \pi_2 P_{21} - \pi_1 = 0$

$\pi_0 P_{01} + \pi_1 (P_{11}-1) + \pi_2 P_{21} = 0$

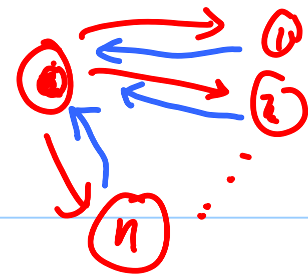
Why $\pi P = \pi \Rightarrow \pi(P-I)=0$

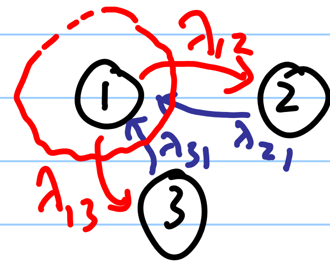
$$\pi_i \sum_{j\neq i} \lambda_{ij} = \sum_{j\neq i} \pi_j \lambda_{ji}$$

$$\sum_i \pi_i = 1$$

$$\pi \mathbf{Q} = 0$$

$$\pi \mathbf{1} = 1$$





$$\pi_1 (\lambda_{12} + \lambda_{13}) = \pi_2 \lambda_{21} + \pi_3 \lambda_{31}$$

$$-(\lambda_{12} + \lambda_{13})\pi_1 + \pi_2 \lambda_{21} + \pi_3 \lambda_{31} = 0$$

$$(\pi_1 \ \pi_2 \ \pi_3) \cdot \begin{bmatrix} -(\lambda_{12}+\lambda_{13}) \\ \lambda_{21} \\ \lambda_{31} \end{bmatrix} = 0$$

$$(\pi_1 \ \pi_2 \ \pi_3) \cdot \underbrace{\begin{bmatrix} -(\lambda_{12}+\lambda_{13}) & \lambda_{12} & \lambda_{13} \\ \lambda_{21} & -(\lambda_{21}+\lambda_{23}) & \lambda_{23} \\ \lambda_{31} & \lambda_{32} & -(\lambda_{31}+\lambda_{32}) \end{bmatrix}}_{Q} = 0$$