Note Title 9/10/2013

$X \sim N(1.5, 3.25^2)$ Q_2: $P(X > 2.35)$?

define: $Z = \dfrac{X - 1.5}{3.25} \sim N(0,1)$

$P(X > 2.35) = 1 - P(X \leq 2.35)$

$X \leq 2.35$

$\Rightarrow 3.25 Z + 1.5 \leq 2.35$

$\Rightarrow Z \leq 0.26$

$= 1 - P(Z \leq 0.26)$

$= 1 - 0.618 = 0.382$

$X \sim B(4, 0.6)$

$P(X=2) = \binom{4}{2} 0.6^2 (1-0.6)^2$

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r.v. X_i : lifetime of battery i

r.v. R : overall time of 25

$R = X_1 + X_2 + \cdots + X_{25}$

$\mu = 40$, $\sigma = 20$, $n = 25$

r.v. $Y = \dfrac{R - n\mu}{\sigma\sqrt{n}} \sim N(0,1)$

$Y = \dfrac{R - 1000}{100}$

Q : $P(R > 1100)$?

$P(R > 1100) = 1 - P(R \leq 1100) = 1 - P(Y \leq 1) = 1 - 0.841 = 0.159$

r.v. X : # of bad reaction persons among 2000 persons

$X \sim B(2000, 0.001) \sim$ Poisson $\lambda = n \cdot p = 2$

$P(X=k) = e^{-\lambda} \frac{\lambda^k}{k!}$ $\quad P(X=3) = \frac{2^3}{3!} \cdot e^{-2} = 0.18$

Q_2 : $P(X>2) = 1 - P(X \le 2) = 1 - [P(X=0) + P(X=1) + P(X=2)]$

r.v. Y = # of trials until a bad reaction

Q_3 : $E[Y] = \frac{1}{p} = 1000$

$Y = \max(X_1, X_2, \cdots, X_n)$

$Z = \min(X_1, X_2, \cdots, X_n)$

Q: $P(Y \le t)$ $\quad X_i \sim X \quad P(X \le t)$

$P(Z \le t)$

$P(Y \le t) = P(X_1 \le t, X_2 \le t, \cdots X_n \le t) = P(X_1 \le t) \cdot P(X_2 \le t) \cdots P(X_n \le t)$

$= P(X \le t)^n$

$P(Z \le t) = 1 - P(Z > t) = 1 - P(X_1 > t, X_2 > t, \cdots X_n > t)$

$= 1 - P(X > t)^n = 1 - [1 - P(X \le t)]^n$

$$\underbrace{0\,0\,0\,1}_{Y_1}\,\underbrace{0\,0\,0\,0\,1}_{Y_2}\,\underbrace{0\,0\,1}_{Y_3}\,\underbrace{0\,1}_{Y_4}$$

halt 3 flip $\begin{cases} 0\,1,1 \\ 1,0\,1 \end{cases}$

r.v. Y_i : # of trials until head in cycle i $\sim$ Geometric $[1, \infty]$

r.v. N = # of heads

$$P(N=n) = P(Y_1 \geq 3, Y_2 \geq 3, \cdots Y_{n-1} \geq 3, Y_n \leq 2)$$

$$= P(Y_1 \geq 3) \cdot P(Y_2 \geq 3) \cdots \cdot P(Y_{n-1} \geq 3) \cdot P(Y_n \leq 2)$$

$$P(Y \leq 2) = P(Y=1) + P(Y=2) = p + (1-p) \cdot p = 2p - p^2$$

$$P(Y \geq 3) = 1 - P(Y < 3) = 1 - P(Y \leq 2) = (1-p)^2$$

$$P(N=n) = (1-p)^{2n-2} \cdot (2p - p^2)$$

$$E[N] = \sum_{n=2}^{\infty} P(N=n) \cdot n = p(2-p) \cdot \sum_{n=2}^{\infty} n \cdot (1-p)^{2(n-1)} \longrightarrow \sum_{n=2}^{\infty} n \alpha^{n-1}$$

define $\alpha = (1-p)^2$

$$S = \sum_{n=2}^{\infty} n\alpha^{n-1} \qquad \alpha = (1-p)^2 = 1 + p^2 - 2p$$

$$S = 2\alpha + 3\alpha^2 + 4\alpha^3 + 5\alpha^4 + \cdots$$

$$\alpha S = 2\alpha^2 + 3\alpha^3 + 4\alpha^4 + \cdots$$

$$\Rightarrow (1-\alpha)S = \alpha + \underbrace{\alpha + \alpha^2 + \alpha^3 + \alpha^4 + \cdots}_{\to \frac{\alpha}{1-\alpha}}$$

$$= \alpha + \frac{\alpha}{1-\alpha}$$

$$\Rightarrow S = \frac{\alpha(1-\alpha)+\alpha}{(1-\alpha)^2} = \frac{2\alpha-\alpha^2}{(1-\alpha)^2}$$

$$E[N] = p(2-p)\cdot\frac{2\alpha-\alpha^2}{(1-\alpha)^2} = p(2-p)\cdot\frac{2(1-p)^2-(1-p)^4}{(2-p)^2\cdot p^2}$$

$$= \frac{(1-p)^2\left[2-(1-p)^2\right]}{(2-p)\cdot p}$$