# CDA6530: Performance Models of Computers and Networks (Fall 2012)

## Homework 2: Random processes and M/M/* queue

**1.** Jobs arrive at a computer center following a Poisson process with rate $\lambda$. The computer center has two computers called A and B. Each computer has a service rate of $\mu$. We split the arriving work between the two computers using one of the following two splitting rules:

Rule 1: Each arriving job flips a fair coin to decide which computer queue to join; let's say heads it goes to computer A and tails it goes to computer B.

Rule 2: Assuming the jobs are numbered in sequence, the even numbered jobs are directed to computer A and the odd numbered jobs are directed to computer B.

Clearly, Rule 1 is randomized and Rule 2 is deterministic.

(a). What is the distribution of the inter-arrival time for computer A under rule 1 and rule 2, respectively. Please give the distribution pdf function.

(b). What is the average response time for an arrival job under rule 1?

**2.** Consider an M/M/1 system with parameters $\lambda$ and $\mu$ in which exactly two customers arrive at each arrival instant (i.e, after exponentially distributed time with rate $\lambda$, two customers will arrive together).

(a) Under what condition is the system stable?

(b) Draw a state transition rate diagram for the system.

(c) If the arrival follows Poisson process with rate $2\lambda$ (each arrival only arrives one customer). Draw the state transition diagram.

**3.** A telephone system consisting of two lines services three callers. Each caller makes calls that are exponentially distributed in length, with mean $1/\mu$. If both lines are in service by two callers and the third one requests service, the third caller will be blocked. A caller whose previous attempt to make a call was successful (the caller was not blocked) has an exponentially distributed time before attempting the next call, with rate $\lambda$. A caller whose previous call attempt was blocked is impatient and tries to call again at twice that rate ($2\lambda$), also according to exponential distribution. The callers make their calls independent of one another. The objective is to determine the average number of calls in progress.

(a). Draw the state transition diagram for this system. Define all states carefully, and clearly label the nodes and arcs of the diagram.
(b). Give the rate generator matrix (infinitesimal generator Q) for this system, being sure to relate the rows (and columns) to their associated states.

**4. (discrete-time machine-repairmen problem)** A building has a vending area that contains three vending machines. Each of the machines can be in one of three states: working, broken, or in repair. Time is divided into fixed-size steps. A machine that is working at the beginning of a time step is working at the end of the time step with probability 0.7 or is broken with probability 0.3.

The building concessionaire employees one repairman, who can only work on one machine at a time. If the repairman is working on a machine during a time step, the machine remains in repair at the end of the time step with probability 0.4 or is working again with probability 0.6. If the repairman was idle during a time step or finishes working on a machine (it goes to state working) at the end of a time step, and there is another machine in state "broken" at the end of that time step, he immediately begins working on the machine at the beginning of the next time step. If more than one machine is broken when the repairman is ready to start work on another machine, he selects a broken machine at random.

(a). Define an appropriate (and minimal) set of states and draw the state transition diagram for the Markov chain with these states. Be sure to label all nodes and edges in the diagram clearly.
(b). Find the average number of machines that are working during a time step.
(c). Write the single-step transition matrix
(d). Find the steady state probabilities.