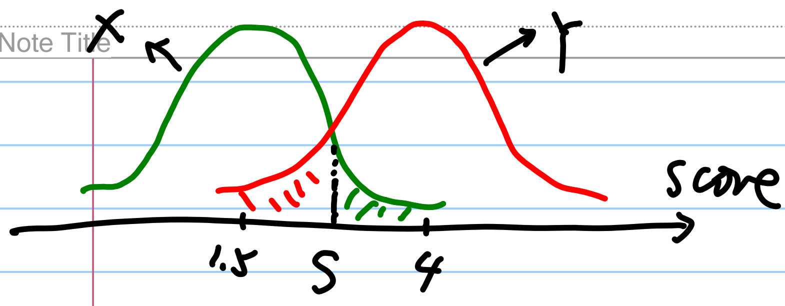
Note Title 9/6/2012

CDA6530 lecture #6

Model :



r.v. X: score of normal

r.v. Y: score of spam

$X \sim N(1.5, \sigma^2)$ $\quad Y \sim N(4, 1)$

Q_1: → S? such that $P(Y \geq S) = 0.95$

$P(Y \geq S) = 0.95$ $\quad$ define r.v. $Z = \frac{Y-4}{1} \sim N(0,1)$

$\Rightarrow P(Z \geq S-4) = 0.95$ $\quad$ since $P(Z \leq -1.65) = 0.05$

so $S - 4 = -1.65 \Rightarrow S = 2.35$

Q_2: $P(X > S)$? $\quad P(X \leq 1.9) = 0.9$

define $Z = \frac{X - 1.5}{\sigma} \sim N(0,1)$ $\quad P\left(Z \leq \frac{1.9-1.5}{\sigma}\right) = 0.9$

$0.4/\sigma = 1.3 \Rightarrow \sigma = 0.308$ $\quad X = \sigma Z + 1.5$

$P(X > 2.35) = P\left(Z > \frac{2.35 - 1.5}{0.308}\right) = P(Z > 2.76) = 1 - P(Z \leq 2.76) = 0.003$

false positive rate $= P(X > s)$

false negative rate $= P(Y < s)$

$p = \frac{3}{5}$ $\quad P(X = 2)$? $\quad P(X=2) = \binom{4}{2} \cdot \left(\frac{3}{5}\right)^2 \cdot \left(1 - \frac{3}{5}\right)^2$

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r.v. X_i : lifetime of battery i $\quad \mu = 40, \quad \sigma = 20$

r.v. Y : lifetime of 25 batteries

$Y = X_1 + X_2 + \cdots + X_{25}$ $\quad$ Q: $P(Y > 1100)$?

$Y \sim N(n\mu, n\sigma^2)$ $\quad Y \sim N(40 \times 25, \; 100^2)$, $\; Z = \frac{Y - 1000}{100}$

$P(Y > 1100) = P(100Z + 1000 > 1100) = P(Z > 1) = 1 - P(Z \leq 1)$

$= 1 - 0.841 = 0.159$

X: # of persons have bad reaction among 2000 persons $\quad p = 0.001$

$\longrightarrow X \sim B(\overset{n}{2000}, \overset{p}{0.001})$

Q_a: $P(X=3)$

treat X as Poisson distr. $\quad \lambda = n \cdot p = 2$

$$P(X=k) = e^{-\lambda}\frac{\lambda^k}{k!} \qquad P(X=3) = e^{-2}\frac{2^3}{3!} = 0.18$$

Q_b: $P(X>2) = P(X=3) + P(X=4) + \dots + P(X=2000)$

$= 1 - P(X \le 2) = 1 - P(X=0) - P(X=1) - P(X=2)$

Q_c $E[Y]$ $\quad Y$: # of trials until a bad reaction

$E[Y] = \frac{1}{p} = \frac{1}{0.001} = 1000$

- $Y = \max(X_1, X_2, \cdots, X_n)$
- $Z = \min(X_1, X_2, \cdots, X_n)$

Q_1 : $P(Z \le t_1)$

Q_2 : $P(Y \le t_2)$

$p(AB) = p(A) \cdot p(B)$

known $p(X \le x)$

$$P(Z \le t_1) = 1 - P(Z > t_1)$$
$$= 1 - P(X_1 > t_1, X_2 > t_1, X_3 > t_1, \cdots X_n > t_1)$$
$$= 1 - p(X_1 > t_1) \cdot p(X_2 > t_1) \cdots \cdot p(X_n > t_1)$$
$$= 1 - [1 - p(X \le t_1)]^n$$

Q_2 : $$P(Y \le t_2) = P(X_1 \le t_2, X_2 \le t_2, \cdots X_n < t_2)$$
$$= P(X \le t_2)^n$$