

CDA 6530 Lecture #5

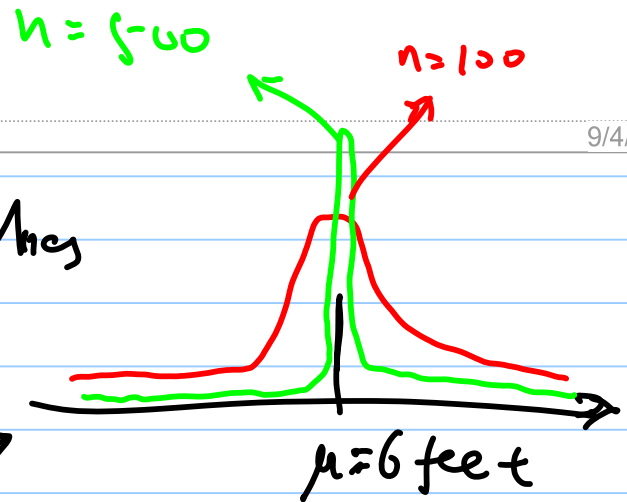
Note Title

9/4/2012

$$\bar{X}_1 = 6 \text{ feet}, \quad \bar{X}_2 = 5 \text{ feet } 11 \text{ inches}$$

$$\bar{X}_3 = 6 \text{ feet } 2 \text{ inches}$$

$$\bar{x} \sim N\left(\mu, \frac{\sigma^2}{n}\right) \rightarrow$$



$$P\left(\sum_{i=1}^{10} X_i > 7\right)$$

$$\text{define } Y \equiv \sum_{i=1}^{10} X_i$$

$$E[X_i] = \frac{1}{2} \rightarrow \mu \quad \text{Var}[X_i] = \frac{1}{12} \rightarrow \sigma^2$$

$$\sqrt{10} \sigma = \frac{1}{\sqrt{12}} \sqrt{10} = \sqrt{\frac{5}{6}}$$

$$Z = \frac{Y - 10\mu}{\sigma \sqrt{10}} \sim N(0, 1) \Rightarrow Y = \sigma \sqrt{10} Z + 10\mu$$

$$\begin{aligned} P(Y > 7) &= P(\sigma \sqrt{10} Z + 10\mu > 7) = P\left(Z > \frac{7 - 10\mu}{\sigma \sqrt{10}}\right) \\ &= P(Z > 2/0.913) = P(Z > 2.19) = 1 - P(Z \leq 2.19) = 0.014 \end{aligned}$$

$$\rightarrow p(X=1, Y=2)$$

□ $p(1,1)=0.5$, $p(1,2)=0.1$, $p(2,1)=0.1$, $p(2,2)=0.3$

□ Q: Calculate the pmf of X given that Y=1

↳ $p(X=1|Y=1)$? $p(X=2|Y=1)$?

$$p(A|B) = \frac{p(AB)}{p(B)}$$

$$p(X=1|Y=1) = \frac{p(X=1, Y=1)}{p(Y=1)}$$

$$= \frac{0.5}{0.6} = 5/6$$

$$p(X=2|Y=1) = \frac{p(X=2, Y=1)}{p(Y=1)} = \frac{0.1}{0.6} = 1/6$$

$$\begin{aligned} p(Y=1) &= p(Y=1, X=1) + p(Y=1, X=2) \\ &= p(1,1) + p(2,1) \\ &= 0.6 \end{aligned}$$

$$\square Y = X_1 + X_2 + \dots + X_N$$

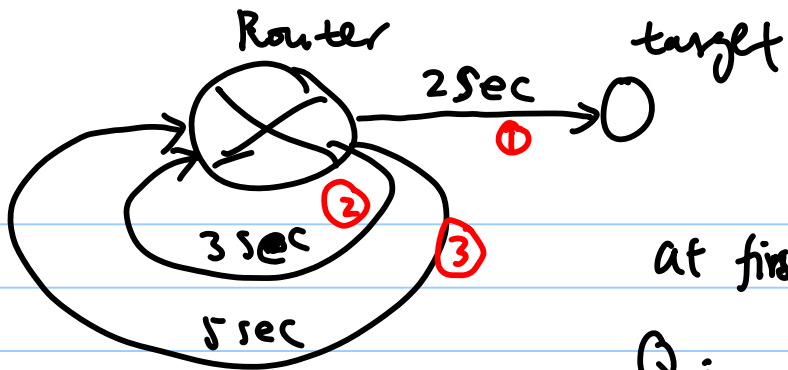
$$\text{if fix } N=n, \quad Y = \sum_{i=1}^n X_i$$

$$E[Y|N=n] = E\left[\sum_{i=1}^n X_i\right] = n \cdot E[X]$$

$$E[Y] = E_N[E[Y|N=n]] = E_N[n \cdot E[X]] = E[X] \cdot E[N]$$

$$Y = X_1 + X_2 + \dots + X_{2N}$$

$$E[Y] = E_N[E[Y|N=n]] = E_N[2n \cdot E[X]] = 2 E[N] \cdot E[X]$$



r.v. T : time to reach target

r.v. F : path of a packet going out at first step $\{1, 2, 3\}$

Q: $E[T]$?

$$E[T|F=1] = 2, \quad E[T|F=2] = 3 + E[T] \quad \checkmark$$

$$E[T|F=3] = 5 + E[T] \quad \checkmark$$

law of total prob:

$$E[T] = E[T|F=1] \cdot P(F=1) + E[T|F=2] \cdot P(F=2) + E[T|F=3] \cdot P(F=3)$$

$$= \frac{2 + 3 + E[T] + 5 + E[T]}{3} = \frac{10 + 2E[T]}{3}$$

$$\Rightarrow E[T] = 10 \text{ sec}$$

