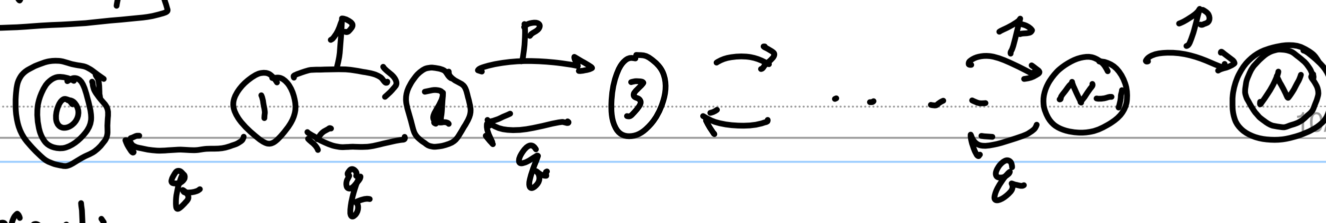
Note Title 10/18/2012

CDA6530, lecture 17

$X_n$




absorbing state

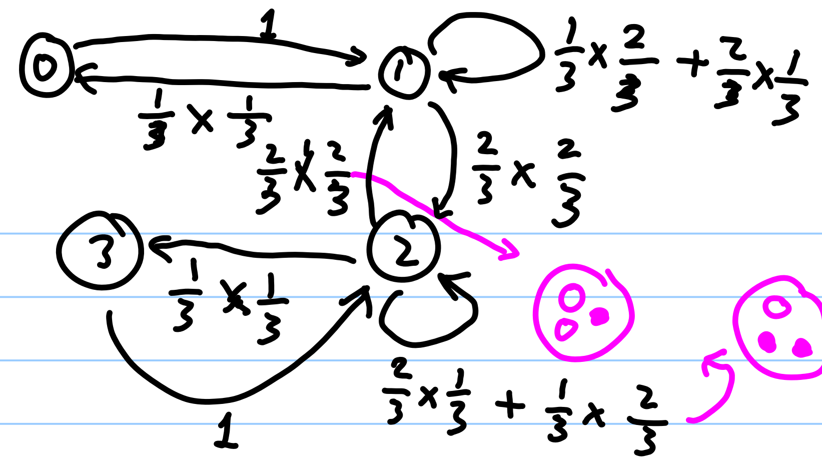
$$P_1 < P_2 < P_3 \cdots < P_{N-1}$$

$$P_0 = 0, \quad P_N = 1$$

- $P_i = p\cdot P_{i+1} + q\cdot P_{i-1}$

$$\begin{cases} i=1 \Rightarrow & P_1 = pP_2 + qP_0 = pP_2 \\ i=2 \Rightarrow & P_2 = pP_3 + qP_1 \\ \vdots & \vdots \\ i=N-1 \Rightarrow & P_{N-1} = pP_N + qP_{N-2} = p + qP_{N-2} \end{cases}$$

$X_n = \{0, 1, 2, 3\}$




partial checking

$$\sum P(\text{jumping out}) = 1$$

check ② : $\frac{4}{9} + \frac{1}{9} + \left(\frac{2}{9} + \frac{2}{9}\right) = 1$

$$\begin{cases} \pi P = \pi \\ \pi \mathbf{1} = 1 \end{cases}$$

$$P = \begin{array}{c c} & \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1/9 & 4/9 & 4/9 & 0 \\ 0 & 4/9 & 4/9 & 1/9 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{1} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\pi(P - I) = 0$$

$$(\pi_1, \pi_2, \pi_3, \pi_4) \underbrace{\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 4/9-1 & 4/9 & 0 \\ 1 & 4/9 & 4/9-1 & 1/9 \\ 1 & 0 & 1 & 0-1 \end{bmatrix}}_{A} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}}_{B}$$

$$\pi = B \cdot A^{-1}$$

$$
\begin{aligned}
&P[\{\text{Nobody is admitted to see the docotr in the } 1^{\text{st}} \text{ hr}\}] \\
&= P[\{\text{At most 2 patient arrive in first 60 mins}\}] \\
&= P[X(t) \leq 2 \text{ over } [0,60]] \\
&= P[X(60) \leq 2] \\
&= P[X(60)=0] + P[X(60)=1] + P[X(60)=2] \\
&= e^{-60/10} + \left(\frac{60}{10}\right) e^{-60/10} + \frac{1}{2}\left(\frac{60}{10}\right)^2 e^{-60/10} \\
&= e^{-6}(1+6+18) \\
&= 0.062.
\end{aligned}
$$

$= P\{\text{the 3rd patients comes} > 60\text{min}\}$

3rd patient arrival time $T_3$

$T_3 \sim$ 3rd-order Erlang distr.

$P(T_3 > 60)$

$= 1 - F_{T_3}(60)$

$= \sum_{n=0}^{2} \frac{1}{n!} e^{-\lambda 60} \cdot (\lambda 60)^n$