Note Title 9/8/2011

(2)

0 0 0 1 0 0 0 0 1 0 0 1 0 1

Y_1, Y_2, Y_3, Y_4

Y_i = # of trials in cycle i

(1) 11

(3) 001 | 001 |

$P(N=n) = P(Y_1 \geq 3$ & $Y_2 \geq 3 \cdots$ & $Y_n \leq 2)$

$Y_i \sim$ Geometric with p

$P(N=n) = P(Y_1 \geq 3) \cdot P(Y_2 \geq 3) \cdots P(Y_n \leq 2)$

$P(Y \leq 2) = P(Y=1) + P(Y=2) = p + (1-p) \cdot p = 2p - p^2$

$P(Y \geq 3) = 1 - P(Y \leq 2) = 1 - 2p + p^2 = (1-p)^2$

$P(N=n) = (1-p)^{2(n-1)} \cdot (2p - p^2)$

$E[N] = \sum_{n \geq 2}^{\infty} P(N=n) \cdot n = p(2-p) \cdot \sum_{n \geq 2}^{\infty} n(1-p)^{2n-2}$

$$\alpha = (1-p)^2 \quad \hookrightarrow \quad \sum_{n=2}^{\infty} n\alpha^{n-1} = S$$

$$S = 2\alpha + 3\alpha^2 + 4\alpha^3 + 5\alpha^4 \cdots\cdots$$

$$\alpha S = 2\alpha^2 + 3\alpha^3 + 4\alpha^4 + 5\alpha^5 \cdots\cdots$$

$$(1-\alpha)S = \alpha + \alpha + \alpha^2 + \alpha^3 + \cdots\cdots = \alpha + \frac{\alpha}{1-\alpha}$$

$$
\begin{aligned}
P_n(t+\Delta t) &= P_{n-1}(t)\lambda\Delta t + P_n(t)(1-\lambda\Delta t) + o(\Delta t) \\
P_n(t+\Delta t) - P_n(t) &= P_{n-1}(t)\lambda\Delta t - P_n(t)\lambda\Delta t + o(\Delta t) \\
\frac{P_n(t+\Delta t) - P_n(t)}{\Delta t} &= P_{n-1}(t)\lambda - P_n(t)\lambda + \frac{o(\Delta t)}{\Delta t}
\end{aligned}
$$

① $(n-1)$ arrived by t, 1 arrives $(t, t+\Delta t)$

② n arrived by t, 0 arrives in $(t, t+\Delta t)$

- $dP_0(t)/dt = -\lambda P_0(t)$ $\rightarrow$ $P_0(t) = c\, e^{-\lambda t}$ $\quad p_0(0) = 1$

  $\Rightarrow c = 1$