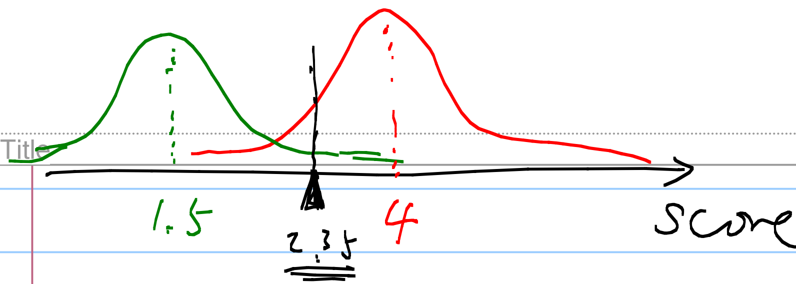
Note Title 9/6/2011

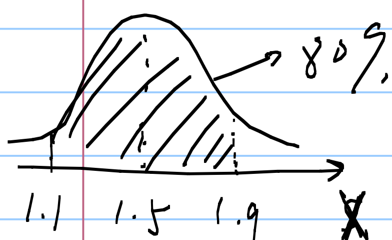


Model:

X : score of normal

Y : score of spam

$X \sim N(1.5, \sigma^2)$ $\qquad$ $Y \sim N(4, 1)$



$P(X \le 1.9) = 0.9$

define $Z_1 = \dfrac{X - 1.5}{\sigma} \sim N(0,1)$

$P\left(Z_1 \le \dfrac{1.9-1.5}{\sigma}\right) = 0.9$ $\qquad$ $\because \Phi(1.3) = 0.9$

$\therefore \dfrac{1.9-1.5}{\sigma} = 1.3$

$\Rightarrow \sigma = 0.308$

define S as threshold $\qquad$ $P(Q_1) \Rightarrow P(Y \le S) = 0.05$

$Z_2 = \dfrac{Y-4}{1} \sim N(0,1)$ $\qquad$ $P(Y \le S) = P(Z_2 \le S-4) = 0.05$

$\because \Phi(-1.65) = 0.05 \Rightarrow S - 4 = -1.65 \Rightarrow S = 2.35$

Q_2: $P(X > 2.35)$?

we know $X \sim N(1.5, 0.308^2)$

$$P(X \le 2.35) = P\left(Z_1 \le \frac{2.35-1.5}{0.308}\right) = P(Z_1 \le 2.76)$$

$$= 1 - P(Z \le -2.76) = 1 - 0.029 \Rightarrow P(X > 2.35) = 0.029$$

X: # of white balls

$$P(X=2) = \binom{4}{2}\left(3/5\right)^2\left(1-3/5\right)^2$$

X_i: life time for battery i $\qquad X_i \sim N(\mu, \sigma^2)$

Y: total life time of 25 batteries

$$Y = X_1 + X_2 + \cdots + X_{25} \qquad P(Q) = P(Y \ge 1100)$$

$$Y \sim N(n\mu, n\sigma^2) \rightarrow Y \sim N(1000, 100^2), \quad Z = \frac{Y-1000}{100}$$

$$P(Y \ge 1100) = P\left(Z \ge \frac{1100-1000}{100}\right) = P(Z \ge 1) = 1 - P(Z \le 1) = 1 - 0.841$$

$$= 0.159$$

X: # of person having bad reaction

X ~ Poisson distr. $\lambda = np = 2000 \times 0.001 = 2$

$$P(X=k) = e^{-\lambda}\frac{\lambda^k}{k!} \qquad P(X=3) = e^{-2} \cdot \frac{2^3}{3!} = 0.18$$

$$P(Q2) = 1 - P(X \le 2) = 1 - [P(X=0) + P(X=1) + P(X=2)]$$

Y: # of trials before we succeed once

Y ~ Geometric $\quad E[Y] = \frac{1}{p} = 1000$

$Q_2 \leftarrow$ ▫ $Y = \max(X_1, X_2, \cdots, X_n)$

$Q_1 \leftarrow$ ▫ $Z = \min(X_1, X_2, \cdots, X_n)$

$P(AB) = P(A) \cdot P(B)$

$$P(Y \le x) = P(X_1 \le x \ \& \ X_2 \le x \cdots \ \& \ X_n \le x)$$

$$= P(X \le x)^n$$

$$P(Z \le x) = 1 - P(Z > x)$$

$$= 1 - P(X_1 > x \ \& \ X_2 > x \ \& \cdots \& \ X_n > x)$$

$$= 1 - P(X_1 > x) \cdot P(X_2 > x) \cdots P(X_n > x)$$

$$= 1 - [1 - P(X \le x)]^n$$