No 10/20/2011

$$G_X(z) = \sum_{k=0}^{\infty}(1-p)p^k z^k = \frac{1-p}{1-pz},$$

$$\Leftarrow \quad (1-p)\sum_{k=0}^{\infty}(pz)^k = (1-p)\cdot\frac{1}{1-pz} \qquad \text{if } pz<1$$

$$\boxed{\sum_{k=0}^{\infty}\frac{\alpha^k}{k!} = e^{\alpha}}$$

$$G_X(z) \equiv E[z^X] = \sum_{k=0}^{\infty} p_k z^k \qquad \frac{dG_X(z)}{dz} = \sum_{k=1}^{\infty} k p_k z^{k-1}$$

let $z=1$

$$\sum_{k=1}^{\infty} k p_k = E[X]$$

$$\left.\frac{d^2 G_X(z)}{dz^2}\right|_{z=1} = \left.\sum k(k-1)p_k z^{k-2}\right|_{z=1} = \underbrace{\sum k^2 p_k}_{E[X^2]} - \underbrace{\sum k p_k}_{E[X]}$$

$$(\lambda+\mu)\pi_i = \lambda\pi_{i-1}+\mu\pi_{i+1}, \quad i=1,\ldots$$
$$\lambda\pi_0 = \mu\pi_1$$

$$G_X(z) \equiv E[z^X] = \sum_{k=0}^{\infty} p_k z^k$$

z^i

$$(1+\rho)\pi_i z^i = \rho\pi_{i-1}\cdot z^{i-1}\cdot z + \frac{\pi_{i+1}\cdot z^{i+1}}{z} \qquad i=1,2,3,\ldots$$

adds all i

$$\begin{cases} (1+\rho)\sum_{i=1}^{\infty}\pi_i z^i = \rho z \sum_{i=0}^{\infty}\pi_i z^i + \frac{1}{z}\sum_{i=2}^{\infty}\pi_i z^i \\ (1+\rho)\pi_0 = \pi_1 + \pi_0 \end{cases}$$

$$\pi_2 z^2 + \pi_3 z^3 + \cdots = G_N(z) - \pi_0 - \pi_1 z$$

$$(1+\rho)\underbrace{\sum_{i=0}^{\infty}\pi_i z^i}_{G_N(z)} = \rho z G_N(z) + \frac{1}{z}\left(G_N(z)-\pi_0\right) + \pi_0$$

$$F_X^*(s) \equiv E[e^{-sX}] = \int_0^{\infty} f_X(x)e^{-sx}\,dx$$

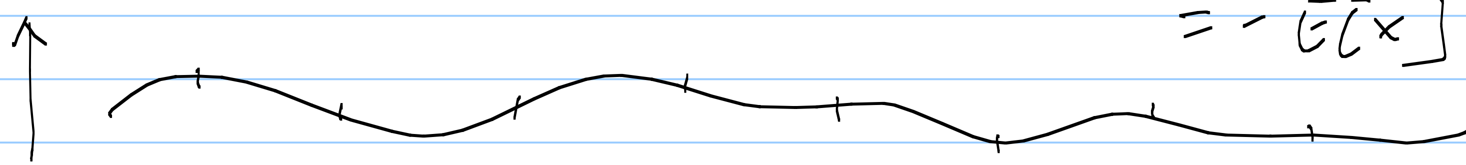
$$E[X] = \int_{-\infty}^{\infty} x \cdot f_x(x)\,dx \qquad E[H(X)] = \int_{-\infty}^{\infty} f_x(x) \cdot H(x)\,dx$$

$$E[X^2] = \int_{-\infty}^{\infty} f_x(x) \cdot x^2\,dx$$

$$F_X^*(s) \equiv E[e^{-sX}] = \int_0^{\infty} f_X(x) e^{-sx}\,dx$$

$$\left.\frac{dF^*(s)}{ds}\right|_{s=0} = -\int_0^{\infty} f_x(x)\, x \cdot e^{-sx}\,dx \Big|_{s=0} = -\int_0^{\infty} f_x(x) \cdot x \cdot 1 \cdot dx$$

$$= -E[X]$$



- $X_i(t) = (U-0.5) + (X_{i-1}(t-1) + X_{i+1}(t-1)) / 2$

```
Simul_N = 1000; n=100; X = ones(n,1);
for k=1:Simul_N,
    U = rand(n,1);
    X(1) = (U(1) - 0.5) + X(2)/2;
    for i=2:n-1,
        X(i) = (U(i) - 0.5) + (X(i-1) + X(i+1)) / 2;
    end
    X(n) = (U(n) - 0.5) + X(n-1) / 2;
end
```

X = ones(n,1)

X_d == X ;

$X(i) = U(i) - 0.5 + \dfrac{(X\_d(i-1) + X\_d(i+1))}{2}$